ICDEA 2022

27th International Conference on Difference Equations and Applications

18-22 July 2022, Paris-Saclay, France

https://icdea2022.sciencesconf.org/

Organized by L2S -- Laboratory of Signals and Systems

Laboratoire Signaux & Systèmes

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I. Welcome Address from ICDEA Organizing Committee

Dear participants,

a very warm welcome to the 27th International Conference on Difference Equations and Applications (ICDEA), that takes place in Gif-sur-Yvette, France, at CentraleSupélec, at the heart of Paris-Saclay University from 18 to 22 of July 2022.

ICDEA 2022 is a major event of the International Society of Difference Equations (ISDE), and brings together researchers and scientists from around the world, to present, discuss and offer solutions in the fields of Difference Equations, Discrete Dynamical Systems, and their applications to various sciences as mathematical biology, epidemiology, evolutionary game theory, economics, physics, and engineering.

Despite the uncertain situation due to the pandemics, 205 researchers have pre-registered and 170 will participate to ICDEA 2022, both physically and online. The submitted abstracts have been reviewed and will be presented in 50 Regular Sessions or Special Sessions and as Posters. Furthermore, 12 Plenary Sessions will describe both theoretical-oriented and industrial-oriented applications.

During the week 18-22 July 2022, the same location will welcome the 18th IFAC Workshop on Control Applications and Optimization (IFAC CAO 2022). The participants to ICDEA 2022 will have the opportunity to connect and discuss with the participants to this IFAC Workshop dedicated to latest developments in Optimal Control Theory and the Optimization-based design in automation and decision-making processes.

We remain looking forward to welcome you in Gif-sur-Yvette
Kind regards,

Sorin Olaru – General Chair

Alessio Iovine – Co-Chair of the Organizing Committee

II. A letter from the President of ISDE

On Behalf of the International Society of Difference Equations (ISDE), it's with tremendous pleasure that I welcome you all to our first hybrid conference, the 27th International Conference on Difference Equations and Applications (ICDEA 2022) which will be held at CentraleSupélec in Gif-sur-Yvette in Paris region, July 18-22, 2022.

Building on the success of previous meetings, I look forward to learning the latest results from top mathematicians and academic scientists on topics related to the areas of difference equations, discrete dynamical systems and applications to science, engineering, and economics. We are indeed in a time of great innovation in these areas, so come and enjoy the research, and report your results.

The program has already shaped up to be excellent, and the networking opportunities will be indeed outstanding. The backdrop of the beautiful and historic city of Paris will add to the pleasure of the meeting and provide lasting memories beyond the science.

I add my best wishes for a successful and fruitful conference and my thanks to all the organizers for their great efforts and hard work.

I will see you soon in Paris region, France!

Saber Elaydi
ISDE President

III. Conference Committees

III.1 Organizing Committee

General Chair	
Sorin Olaru	CentraleSupélec, France
Co-Chair	
Alessio Iovine	CNRS - L2S, France
Members	
Carlos Dorea	Federal University of Rio Grande do Norte, Brazil
Guilherme Mazanti	INRIA, France
Florentina Nicolau	ENSEA, France
Laurent Pfeiffer	INRIA, France
Serban Sabau	Stevens Institute of Technology, USA
Adnane Saoud	CentraleSupélec, France
Florin Stoican	Politehnica University, Romania
Junbo Tan	Tsingua University, China
Cristina Vlad	CentraleSupélec, France

III.2 Scientific Committee

Chair	
Jim Cushing	University of Arizona, USA
Members	
Antoine Girard	CNRS- University Paris-Saclay, France
Rene Lozi	Université Côte d'Azur, France
Sorin Olaru	CentraleSupélec - University Paris-Saclay, France
Adina Sasu	West University of Timisoara, Romania
Jianshe Yu	Guangzhou University, China

III.3 Organizing Institutions

International Society of Difference Equations

CentraleSupelec, Paris-Saclay University

Laboratory of Signals and Systems

IV. Sponsors

Industrial Sponsors

RTE, France

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CentraleSupélec Foundation

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V. About ICDEA History

The previous conferences on Difference Equations and Applications were held in:

- 26th, Sarajevo, Bosnia and Herzegovina, July 26-30, 2021,
- 25th, London, UK, June 24-28, 2019,
- 24th, Dresden, Germany, May 21-25, 2018,
- 23rd, Timisoara, Romania, July 24-28, 2017,
- 22nd, Osaka, Japan, July 24-29, 2016,
- 21st, Bialystok, Poland, July 19-25, 2015,
- 20th, Wuhan, Hubei, China, July 21-25, 2014,
- 19th, Muscat, Oman, May 26-30, 2013,
- 18th, Barcelona, Spain, July 24-29, 2012,
- 17th, Trois-Rivières, Quebec, Canada, July 24-29, 2011,
- 16th, Riga, Latvia, July 19-23, 2010,
- 15th,Estoril, Portugal, October 19-23, 2009,
- 14th, Istanbul, Turkey, July 21-25, 2008,
- 12th, Lisbon, Portugal, July 23-27, 2007,
- 11th, Kyoto, Japan, July 24-28, 2006,
- 10th, Münich, Germany, July 25-30, 2005,
- 9th, Los Angeles, California, USA, August 2-6, 2004,
- 8th, Brno, Czech Republic, July 28 - August 1, 2003,
- 7th, Changsha, China, August 12-17, 2002,
- 6th, Augsburg, Germany, July 20 - August 3, 2001,
- 5th, Temuco, Chile, January 2-7, 2000,
- 4th, Poznan, Poland, August 27-31, 1998,
- 3rd, Taipei, Taiwan, 1997,
- 2nd, Veszprém, Hungary, 1995,
- 1st, San Antonio, Texas, USA, 1994.

VI. Plenary Talks

1- Synchronization Patterns and Chimera States in Networks of Coupled Maps

Eckehard Schöll (Technische Universität Berlin, Germany)

Date and Time: Thursday, July 21, 13:30-14:30 (Paris time)

Abstract

Chimera states are an intriguing example of partial synchronization patterns emerging in nonlocally coupled networks of identical chaotic time-discrete maps or time-continuous oscillators. They consist of spatially coexisting domains of coherent (synchronized) and incoherent (desynchronized) dynamics. We show that a plethora of partial synchronization scenarios including chimeras arise for various network topologies like one-dimensional ring networks, quasi-fractal connectivities, two-dimensional lattices, or multi-layer structures, and different dynamical maps. In particular, we study the logistic map, the Hénon map and the Lozi map. By analyzing the dependence of the spatiotemporal dynamics on the range and strength of coupling, we uncover dynamical bifurcation scenarios for the transition from coherence to complete incoherence via chimera states, and review numerical and analytial approaches [1-5].

[1] Omelchenko, I., Maistrenko, Y., Hövel, P. and Schöll, E., Loss of coherence in dynamical networks: spatial chaos and chimera states, Phys. Rev. Lett. 106, 234102 (2011).
[2] Hagerstrom, A. M., Murphy, T. E., Roy, R., Hövel, P., Omelchenko, I. and Schöll, E., Experimental Observation of Chimeras in Coupled-Map Lattices, Nature Phys. 8, 658 (2012).
[3] Semenova, N., Zakharova, A., Schöll, E. and Anishchenko, V. S., Does hyperbolicity impede emergence of chimera states in networks of nonlocally coupled chaotic oscillators?, Europhys. Lett. 112, 40002 (2015).
[4] Winkler, M., Sawicki, J., Omelchenko, I., Zakharova, A., Anishchenko, V. and Schöll, E., Relay synchronization in multiplex networks of discrete maps, Europhys. Lett 126, 50004 (2019).
[5] Shepelev, I. A., Muni, S. S., Schöll, E. and Strelkova, G. I., Repulsive inter-layer coupling induces anti-phase synchronization, Chaos 31, 063116 (2021).

Biography

Eckehard Schöll is a Professor of Theoretical Physics at TU Berlin, a Guest Scientist at the Potsdam Institute for Climate Impact Research, and a Principal Investigator of the Bernstein Center for Computational Neuroscience Berlin. He studied physics at the University of Tübingen (Germany), and holds PhD degrees in mathematics from the University of Southampton (UK, 1978) and in physics from RWTH Aachen (Germany, 1981), and an Honorary Doctorate from Saratov State University (Russia, 2017). He is an expert in the field of nonlinear dynamical systems and head of the group Nonlinear Dynamics and Control. His work pertains to research in the fields of mathematics and physics, particularly semiconductor physics, laser physics, neurodynamics, complex systems and networks, and bifurcation theory. His latest research is also related to topics in biology and the social sciences, e.g. simulation of the dynamics in socioeconomic, physiological, or neuronal networks and power grids. He is one of the forerunners into the research of chimera states. strategic information transmission, spread of disinformation, and deception; security and trust; energy systems; and cyber-physical systems.

2- Two Applications of Discrete Population Models with Hybrid Features

Suzanne Lenhart (University of Tennessee)

Date and Time: Tuesday, July 19, 15:00-16:00 (Paris Time)

Abstract

Discrete models may have hybrid features due to a variety of mechanisms. We will present two examples of hybrid models, one with some mechanisms between time steps and the other with a shorter time scale embedded in the main time scale processes. In our first example, we present mechanistic models featuring within-season harvest timing and level. Between discrete population census times, there are continuous density-dependent mortality rates. We explore optimal control of the harvest level and timing of these population models. Maximizing the yield while minimizing the costs of management can lead to unexpected oscillation patterns in the population. In our second example, we find the relationship between air temperature and emergence success of hatchlings across multiple nesting seasons to better understand the potential impact of climate change on Loggerhead sea turtle populations. We want to investigate the effect of changing hatchling emergence success on the juvenile and adult populations. The results of a statistical model of emergence success feed into the dynamics during the nesting seasons of the eggs and hatchlings in a submodel on daily time scale. The submodel is connected to an age-structure model with two juvenile and one adult classes on a yearly time scale. We illustrate the effect of temperature changes across these life stages.

Biography

Suzanne Lenhart is a Chancellor's professor in the Department of Mathematics at the University of Tennessee, Knoxville. She was a part-time research staff member at Oak Ridge National Laboratory for 22 years. She is a recognized expert in optimal control and ordinary and partial differential equations, with modeling applications to populations, natural resources, invasive species, and diseases. She has authored many journal articles, as well as two texts, *Optimal Control applied to Biological Model* and *Mathematics for the Life Sciences.*

She was the President of the Association for Women in Mathematics in 2001-2003. She received fellow awards from SIAM, AMS, AWM, and AAAS. She was the Associate Director for Education and Outreach of the National Institute for Mathematical and Biological Synthesis for the last 12 years. Lenhart has been the director of Research Experiences for Undergraduates summer programs on her campus for 27 years.

3- Mathematics of Mechanisms (CentraleSupélec Foundation's Plenary Talk)

Franco Blanchini (Udine University, Italy)

Date and Time: Tuesday, July 19, 11:00-12:00 (Paris Time)

Abstract

Systems and control theory have developed many techniques borrowing tools from different branches of mathematics. Interestingly, many of the techniques conceived and routinely used to solve control problems can be quite successfully adapted to solve new relevant problems, both practical and curiosity-driven, in other fields.

In this talk we discuss how the mathematical analysis of systems can be very effective in explaining how mechanisms work, why they work in a certain way and to which extent they perform their task properly even in the presence of perturbations and disturbances. The first part of the talk briefly introduces some preliminary motivating examples of mechanisms, borrowed from other disciplines alien to control theory, to show how a control approach can be very powerful to understand fundamental principles. The second part introduces the definitions of structural versus robust properties, discussing paradigmatic case studies from the literature. These include a discussion about robust stability/instability analysis, presented in an inverse form: "We know this is stable, but why is it so incredibly stable?". Other fundamental concepts such as adaptation, oscillations, bistability and graph loop analysis are considered. The third part discusses application examples from biology and biochemistry, to showcase the potential impact that the mathematical approach of control and system theory, suitably revised, can have in these disciplines and how interdisciplinary research can bring fresh ideas to theorists.

Biography

Franco Blanchini was born on 29 December 1959, in Legnano (Italy). He is the Director of the Laboratory of System Dynamics at the University of Udine. He has been involved in the organization of several international events: in particular, he was Program Vice-Chairman of the conference Joint CDC-ECC 2005, Seville, Spain; Program Vice-Chairman of the Conference CDC 2008, Cancun, Mexico; Program Chairman of the Conference ROCOND, Aalborg, Denmark, June 2012 and Program Vice-Chairman of the Conference CDC 2013, Florence, Italy. He is co-author of the book "Set theoretic methods in control", Birkhauser. He received the 2001 ASME Oil & Gas Application Committee Best Paper Award as a co-author of the article "Experimental evaluation of a High-Gain Control for Compressor Surge Instability", the 2002 IFAC prize Survey Paper Award as the author of the article "Set Invariance in Control - a survey", Automatica, November 1999, for which he also received the High Impact Paper Award in 2017, and the 2017 NAHS Best Paper Award as a co-author of the article "A switched system approach to dynamic race modelling", Nonlinear Analysis: Hybrid Systems, 2016. He has been plenary speaker at the IFAC Joint Conference, Grenoble, February 2013, plenary speaker at the 32nd Benelux Meeting on Systems and Control, and semi-plenary speaker at the IEEE CDC 2020, Jeju, Japan. He has been an Associate Editor for Automatica and for IEEE Transactions on Automatic Control. He has been a Senior Editor for IEEE Control Systems Letters.

4- Symbolic Summation, Linear Difference Equations and Challenging Applications

Carsten Schneider (Johannes Kepler University, Austria)

Date and Time: Wednesday, July 20, 12:00-13:00 (Paris Time)

Abstract

Inspired by Karr's summation algorithm (1981), an advanced summation framework has been developed in the last 20 years that can deal with indefinite and definite multi-sum expressions. The underlying backbone is an algorithmic difference ring theory which enables one to rephrase expressions in terms of indefinite nested sums and products completely automatically to canonical form representations. As a by-product, the computed expressions are built by sums and products that are algebraically independent among each other. Together with general and highly efficient difference ring algorithms for recurrence finding (based on Zeilberger's creative telescoping paradigm) and recurrence solving, one can simplify huge expressions of multi-sums (and related multi-integrals) to certain classes of special functions defined by iterated sums (and integrals). In a nutshell, the developed algorithms for symbolic summation and special functions provide a flexible toolbox that can tackle challenging problems coming, e.g., from combinatorics, special functions, number theory, statistics, or particle physics (a long-term collaboration with DESY, the Deutsches Elektronen-Synchrotron). In this talk the underlying difference ring theory and the available summation tools in this setting will be illustrated by concrete examples that are hard to solve or that could not be solved by other methods so far.

Biography

After his computer science studies at the Friedrich-Alexander-University Erlangen-Nürnberg, **Carsten Schneider** could intensify his scientific interests in the field of computer algebra at the internationally well recognized Research Institute for Symbolic Computation (RISC) of the Johannes Kepler University Linz. There he completed his PhD in 2001, obtained the Habilitation in the field of mathematics in 2008, and serves recently as full processor and vice-director of RISC.

His main research area is symbolic computation: he explores advanced mathematical theories in algebra, in particular in formal rings equipped with ring automorphisms. In this general setting of difference rings, he puts special emphasis on the design of non-trivial algorithms in symbolic summation and finding solutions of linear recurrences. Besides the development of new algorithmic theories, he devotes energy in the design and implementation of user friendly computer algebra packages. Notably, his research has been motivated strongly by concrete and challenging problems, e.g., in number theory (harmonic number identities related to zeta-values and generalizations of them), numerics (Pade approximations, finite element methods), and in particular in combinatorics. In addition, he started 15 years ago an interdisciplinary project with the theory group of the Deutsches Elektronen-Synchrotron (DESY, Zeuthen, Prof. Dr. Johannes Blümlein and Dr. Peter Marquard). In this intensive and fruitful cooperation, they are developing new algorithms for quantum field theories and elementary particle physics. These ongoing large-scale calculations of massive 3--loop Feynman integrals (leading so far to about 80 joint publications) will be used, e.g., for the precision measurements of the Hadron Electron Ring Accelerator (HERA) of the Deutsches Elektronen Synchrotron and of the Large Hadron Collider (LHC) of CERN.

5- On Typical Properties of Lebesgue Measure Preserving Maps in Dimension One

Piotr Oprocha (AGH University of Science and Technology, Poland)
(joint work with Jernej Činč, Jozef Bobok and Serge Troubetzkoy)

Date and Time: Wednesday, July 20, 10:00-11:00 (Paris Time)

Abstract

In this talk I will discuss selected properties of generic continuous maps of the interval and circle which preserve the Lebesgue measure. I will focus on a few natural properties such as entropy, the structure of periodic points, mixing properties, shadowing properties, etc. I will also highlight properties of generic maps compared to other possible dynamical behaviors within maps preserving Lebesgue measure.

In the second part of the talk, I will present the consequences of obtained results for interval maps (not necessarily preserving Lebesgue measure) and two-dimensional dynamics. In this context, I will consider how a parametrized family of maps induces a continuously varying family of one-dimensional attractors in the unit disc. As a possible tool for distinguishing the obtained embeddings, I will refer to the prime ends rotation number on the circle of prime ends associated with these attractors.

The third part discusses application examples from biology and biochemistry, to showcase the potential impact that the mathematical approach of control and system theory, suitably revised, can have in these disciplines and how interdisciplinary research can bring fresh ideas to theorists.

Biography

Piotr Oprocha is full Professor of Mathematics and Dean at Faculty of Applied Mathematics at AGH University of Science and Technology, Kraków, Poland. He received Ph.D. from Jagiellonian University, Kraków, Poland in 2005. In 2018 he was awarded the title of professor of mathematics by the President of Poland. From 2002 he is employed at AGH University of Science and Technology, where he was promoted to full professor in 2018. From 2012 to 2020 he was Deputy Dean for Science and became Dean at the Faculty of Applied Mathematics in 2020. He is an expert in the field of low dimensional discrete dynamical systems with main emphasis on topological and symbolic dynamics. He is authors of numerous papers on topological entropy, notions and mixing, qualitative properties of dynamical systems and chaos.

6- Application of Discrete-Time Stage Structured Models to Understand the Dynamics of a Tick Population and Associated Pathogens

Azmy S. Ackleh (University of Louisiana, USA)

Date and Time: Monday, July 18, 11:00-12:00 (Paris Time)

Abstract

We develop a discrete-time model that describes tick-host population dynamics and incorporates the developmental stages for an individual tick. We study the global dynamics of this system and show that if the inherent net reproductive number is greater than one, then the unique interior equilibrium is globally asymptotically stable. We extend the model to a tick-host-pathogen model to describe the spread of a disease in a hard-bodied tick species. We apply the model to two pathogens and numerically study properties of the invasion and establishment of a disease. In particular, we consider the basic reproduction number, which determines whether a disease can invade the tick-host system, as well as disease prevalence and time to establishment in the case of successful disease invasion. Using Monte Carlo simulations, we calculate the means of each of these disease metrics and their elasticities with respect to various model parameters. We then extend the population model to include spatial movement of ticks and their hosts and analyze the dynamics of the model under a nonlinear fecundity term that exhibits an Allee effect. We show that the system has rich dynamics including monostable and bistable dynamics. We incorporate demographic stochasticity into the model and use numerical simulations to understand spatial invasion patterns of ticks as they relate to different dispersal mechanisms.

Biography

Azmy S. Ackleh is the R.P. Authement Eminent Scholar and Endowed Chair in Computational Mathematics and the Dean of the College of Sciences at the University of Louisiana at Lafayette. He received his Ph.D. from the University of Tennessee at Knoxville in 1993 and then joined the Center for Research in Scientific Computation at North Carolina State University as a postdoctoral fellow. In 1995 he joined the Department of Mathematics at the University of Louisiana as an Assistant Professor. In 2000 he became an Associate Professor of Mathematics and in 2003 he became a Full Professor of Mathematics in that Department.

Ackleh's main research interest is in the area of Mathematical Biology. He co-authored over 150 peer-reviewed articles and a book "Classical and Modern Numerical Analysis: Theory, Methods and Practice". He is particularly interested in the development of continuous and discrete models in population ecology and epidemiology and in using theoretical and numerical tools to understand the short-term and long-term behavior of solutions to these models. Particular applications that he has worked on include selection-mutation models, amphibian dynamics and associated diseases, and the impact of toxicants on aquatic animals and the role of evolution to resist such toxicants on the system dynamics.

7- Discrete Dynamic Models in Social Sciences: Strategic Interaction, Rationality, Evolution

Gian Italo Bischi (Università di Urbino Carlo Bo, Italy)

Date and Time: Monday, July 18, 13:30-14:30 (Paris Time)

Abstract

Discrete-time dynamical systems naturally arise in economic and social modelling, because changes in the state of a system occur as a consequence of decisions (event-driven time). Given a characteristic time interval, taken as a unit of time advancement, then the state at the next time period is obtained by the application of a map, i.e. a point transformation defined in a n-dimensional state space into itself.

Indeed, in social sciences rational agents are assumed to choose their actions through an optimal decision process, with complete information about the environment and the other agents' choices. However, humans are sometimes not so rational nor informed, and they behave following adaptive methods, such as learning-by-doing or trial-and-error practices. This leads to replace one-shot optimal decisions with repeated adaptive decisions, in other words a dynamic process that may or may not converge to a rational equilibrium. Moreover, several attracting sets may coexist, so that the step-by-step adaptive process may act as a selection device and a global study of the basins of attraction can gives information about the path dependence, i.e. how the long-run time evolution depends on historical accidents (represented by exogenous shifts of initial conditions). For example, in repeated oligopoly games strategic choices may never settle to a Nash equilibrium, and continue to move around it, following some periodic or chaotic time patterns, or they may even irreversibly depart from it. Sometimes different decision strategies may be adopted by heterogeneous interacting agents, leading to different time adjustment processes characterized by different performances. In these cases, evolutionary games can be used to model populations of players subdivided into fractions adopting different strategies, and considering endogenous switching process driven by payoff gradients (such as replicator dynamics or imitate the better mechanisms). Evolutionary game theory, initially developed in the context of biological systems, considers the behavior of large populations of agents who repeatedly engage in strategic interactions. In the biological interpretation, a population consists of animals which are genetically programmed to use some strategy that is inherited by its offspring, whereas in the economic interpretation the agents are assumed to play the game many times and consciously switch strategies driven by imitation of more successful opponents. The iterated maps that model such time evolutions in discrete time are generally nonlinear and are often noninvertible. Hence their folding and unfolding properties characterize the attractors and their basins respectively.

Biography

Gian-Italo Bischi is full professor of "Mathematics for economic and social sciences" at the department DESP (Department of Economics, Society, Politics) of the University of Urbino (Italy). He authors several articles and books on nonlinear dynamic models and their applications to the description of complex systems, ranging from physics and biology up to economics and social sciences. He mainly uses methods from the qualitative theory of nonlinear dynamical systems, also in connection with game theory and in particular evolutionary games. He also deals with the popularisation and communication of mathematical ideass, with a particular focus on connections between mathematics and other cultural fields, like literature and art.

8- Nonlinear Discrete-Time Dynamics in Port-Hamiltonian Form

Dorothée Normand-Cyrot (CNRS, France)

Date and Time: Monday, July 18, 17:00-18:00 (Paris Time)

Abstract

Port-Hamiltonian structures have a pervasive impact in numerous applied domains enlarging the more traditional mechanical one. These structures are essentially described in the continuous-time domain while in discrete time, a consensus on a specific structure is not yet reached in spite of a rich literature. In this talk we propose a novel description of port-Hamiltonian structures for nonlinear discrete-time dynamics. We start from a novel representation of controlled nonlinear difference equations as two coupled difference and differential equations. On this basis, a novel notion of passivity for discrete-time systems is introduced that serves to describe port-Hamiltonian forms. We show how damping feedback and energy-based control strategies can be designed on these forms. These structures are validated through the description of the associated Dirac structure and showing closedness under power preserving interconnection. To conclude, we investigate how these forms are transformed under time-integration to recover sampled-data port-Hamiltonian structures thanks to a modification of the interconnection and dissipation matrices characterizing the dynamics. Some simulations are presented to illustrate the validity and control performances of these forms.

Biography

Dorothée Normand-Cyrot received the PhD degree in Mathematics and es-Sciences Doctorat d'Etat in Physics from University Paris VII and Paris Sud in 1978 and 1983 respectively. She joined the French National Center of Scientific Research (CNRS) in 1981 at the Laboratory of Signal and Systems (L2S), Paris Saclay, where she was Research Director from 1991 and currently Emeritus from 2021. She is an IEEE Fellow (class 2005) of the Control Systems Society for Contributions to Digital and Discrete-Time Control Systems. Since 1983, she promoted research activity within Europe and in particular with Italy, holding many visiting positions at the University of Rome La Sapienza through a mobility network on Sciences and Technologies for Information and Control and its Applications (AML-STICA) under the auspices of the French-Italian University UFI/UIF. Her research interest is in the field of nonlinear control theory with focus on discrete-time and digital systems; she is author and co-author of about 217 scientific publications, supervisor of nearly 30 PhD students; recipient of the IFAC-Control Engineering Practice Best Paper Award for 2011/13. Among others, she was Director of the CNRS Groupement de Recherche in Control Theory (GdR-A) from 1998 to 2002; Programme Chair of the ECC-91, Grenoble, France; General Chair of the Workshop Perspectives in Control in 1998, General Chair of CIFA2002 "Conference Internationale Francophone d'Automatique" in Nantes, France. Member of several scientific and steering international committees, she was Funding Member of the European Union Control Association EUCA'91 and Editor at Large of the European Journal of Control from 2003.

9- Phase Resetting as a Two-Point Boundary Value Problem

Hinke Osinga (Auckland University, New Zealand)

Date and Time: Thursday, July 21, 10:00-11:00 (Paris Time)

Abstract

Phase resetting is used in experiments with the aim to classify and characterise different neurons by their responses to perturbations away from a periodic bursting pattern. The same approach can also be applied numerically to a mathematical model. Resetting is closely related to the concept of isochrons of a periodic orbit, which are the submanifolds in its basin of attraction of all points that converge to this periodic orbit with a specific phase. Until recently, such numerical phase resets were performed in an ad-hoc fashion, and the development of suitable computational techniques was only started in the last decade or so. We present an approach based on the continuation of solutions to a two-point boundary value problem that directly evaluates the phase associated with the isochron that the perturbed point is located on. We illustrate this method with the FitzHugh–Nagumo model and investigate how the resetting behaviour is affected by phase sensitivity in the system.

Biography

Hinke Osinga is Professor of Applied Mathematics at the University of Auckland, New Zealand. Her research focuses on dynamical systems theory and the development and application of numerical methods for computing invariant manifolds. Her publications, illustrations, animations and outreach activities have made her famous worldwide in the mathematics and arts communities. She received the 2016 Research Award of the New Zealand Mathematical Society and is a Fellow of the Royal Society of New Zealand, Fellow of the New Zealand Mathematical Society, and Fellow of the Society for Industrial and Applied Mathematics. She obtained her PhD from the University of Groningen, the Netherlands, and has held previous positions at the Universities of Bristol and Exeter in the UK, and Cornell University, California Institute of Technology and the Geometry Center in the USA.

10- From Elliptic Curves to Bifurcation Theory: Differentiable Versus Topological Conjugacy (JDEA Best Paper 2012 Talk by Paul Glendinning and Sasha Glendinning)

Paul Glendinning (University of Manchester, UK)

Date and Time: Tuesday, July 19, 13:00-14:00 (Paris Time)

Abstract

Conjugacy describes how two maps may be related via a change of coordinate. In [1] we show that the dynamics induced by a simple geometric construction on elliptic curves (taking the tangents to the curve at a point and mapping to the other intersection of the tangent to the curve or the point at infinity) creates a map topologically conjugate to the chaotic map x -->2-x^2 on [-2; 2]. Moreover, despite the fact that the numerically computed maps obtained in this way can look very different depending on the parameters of the elliptic curve, they are in fact C1-conjugate. Differentiable conjugacies preserve the stability multipliers of periodic orbits, so in chaotic maps with infinitely many periodic orbits, the existence of differentiable conjugacies is rare.

In the talk I will discuss this result (awarded the best paper for JDEA in 2021) and also address questions about topological versus differentiable conjugacy for normal forms of elementary bifurcations (recent work with David J.W. Simpson, Massey, New Zealand).

[1] P. Glendinning and S. Glendinning (2021) Smooth conjugacy of difference equations derived from elliptic curves, J. Di. Equ. & Appl. doi.org/10.1080/10236198.2021.1984441

Biography

Paul Glendinning is Professor of Applied Mathematics at the University of Manchester. He works on bifurcation theory and piecewise smooth dynamics. He was awarded a PhD from the University of Cambridge in 1985, and after post-docs in Warwick and Cambridge he was a lecturer in Cambridge and then moved to chairs at Queen Mary, University of London, and Manchester (UMIST and University of Manchester). He was the founding Head of School in Mathematics during the merger of UMIST and Manchester (2003-2008), and has been Scientific Director of ICMS, Edinburgh (2016-2021). He is currently President of the IMA, one of the UK Learned Societies for mathematics. He was elected a Fellow of the Royal Society of Edinburgh in 2021.

11- Delays, Interconnections and Control. A Guided Tour

Silviu-Iulian Niculescu (CNRS, France)

Date and Time: Thursday, July 21, 17:00-18:00 (Paris Time)

Abstract

To better grasp the heterogeneity of the temporal phenomena in systems' dynamics, knowledge of “past” may appear as essential to derive appropriate models, and there exists a wide variety of processes in nature with such characteristics. In these cases, the use of delays in the representation of such phenomena may help to better understand the underlying mechanisms or of the interactions/coupling with other (eventually spatial) phenomena. In this context, a particular case is represented by the interconnection of dynamical systems. It is well-known that the interconnection of two or more dynamical systems leads to an increasing complexity of the behavior of the global system due to the effects induced by the emerging dynamics (in the presence or not of feedback loops) in strong interactions (sensing, communication) with environment changes. One of the major problems appearing in such interconnection schemes is related to the propagation, transport, and communication delays acting “through” and/or “inside” the interconnections. Such systems are governed by a particular type of differential equations, namely delay-differential equations.

The purpose of this talk is to briefly present some “user-friendly” (frequency-domain) methods and techniques for the analysis and control of the dynamical systems in the presence of constant (pointwise) delays by emphasizing how the delay, seen as a parameter, can affect the exponential stability of the system. As a by-product of such an analysis, the use of the delay as a control parameter will open up interesting perspectives to exploit further. The presentation is as simple as possible, emphasizing the main intuitive ideas to develop theoretical results, and their potential use in practical applications. Various illustrative examples complete the presentation (joint work with Keqin GU, Southern Illinois University at Edwardsville, USA).

Biography

Silviu-Iulian NICULESCU received the B.S. degree from the Polytechnical Institute of Bucharest, Romania, the M.Sc., and Ph.D. degrees from the Institut National Polytechnique de Grenoble, France, and the French Habilitation (HDR) from Université de Technologie de Compiègne, all in Automatic Control, in 1992, 1993, 1996, and 2003, respectively. He is currently Research Director at CNRS (French National Center for Scientific Research), L2S (Laboratory of Signals and Systems), a joint research unit of CNRS with CentraleSupélec and Université Paris-Saclay located at Gif-sur-Yvette.

Dr. Niculescu is a member of the Inria team “DISCO” and he was the head of L2S for a decade (2010-2019). He is author/coauthor of 11 books and of more than 600 scientific papers. His research interests include delay systems, robust control, operator theory, and numerical methods in optimization, and their applications to the design of engineering systems. Since 2017, he is the Chair of the IFAC TC “Linear Control Systems” and he served as Associate Editor for several journals in Control area, including the IEEE Transactions on Automatic Control (2003-2005). IEEE Fellow since 2018, Doctor Honoris Causa of University of Craiova (Romania) since 2016, Founding Editor and Editor-in-Chief of the Springer Nature Series “Advances in delays and dynamics” since its creation in 2012, Dr. Niculescu was awarded the CNRS Silver and Bronze Medals for scientific research and the Ph.D. Thesis Award from Grenoble INP (France) in 2011, 2001 and 1996, respectively. For further information, please visit https://cv.archives-ouvertes.fr/silviu-iulian-niculescu.

12- Stability and Realization of Difference Equations over $\mathbb{Z}$ and $\mathbb{R}$

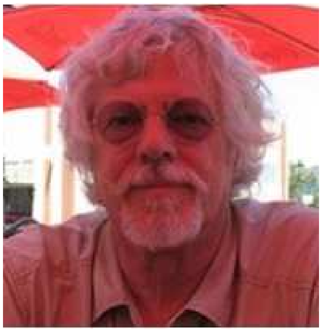
Erik I. Verriest (Georgia Tech, USA)

Date and Time: Friday, July 22, 10:00-11:00 (Paris Time)

Abstract

For LTI (ordinary) difference equations, the Jury (and mapped Routh-Hurwitz) tests provide necessary and sufficient conditions for stability. Our first aim is to show some sufficient conditions of much lower complexity for stability of a subclass of "sparse" systems, and show that this is can be accomplished by a Riccati-equation type condition. These criteria can also be expressed in terms of function-theoretic concepts. We then extend these results to time-variant (delay) difference equations, where problems with wellposedness and causality may occur. Such analyses are instrumental for stability analysis of delay equations and as a singular limit for delay-differential equations. Much of this is joint work with A. Ivanov. The second part of the talk deals with difference equations defined over and $\mathbb{R}$ rather than $\mathbb{Z}$. In particular, the cases with time-varying and state-dependent delay present new problems. First, an appropriate state space for such systems is sought, as this is a prerequisite for precise notions of what is meant by a trajectory and its stability. Then we discuss the infinitesimal generator. This leads then to implicit iterations of the form $x(t) = f(x(t-1), x(t-\tau(x(t)))$, modeling certain systems with state-dependent delay, $\tau(x)$. While here $t \in \mathbb{R}$, we show that a discrete event-structure (i.e., space-time structure) is appropriate for its description, and certain iterated functional equations are instrumental for characterizing solutions. Finally, for nonhomogeneous linear difference equations, i.e., models of discrete input-output systems, we present in part 3 some intricate problems regarding loss of causality and its consequences if the delay is switched. Fixes, known as lossless causalization and forgetful causalization are discussed.

Biography



Erik I. Verriest is the Director of the Mathematical System Theory Laboratory (MAST Lab) at the Georgia Institute of Technology. He was with the Control Systems Laboratory and the Hybrid Computation Centre, Ghent, Belgium in 1973-1974, obtained the Ph.D. degrees from Stanford University and joined the faculty of Electrical and Computer Engineering at the Georgia Institute of Technology in 1980. His present interests are in mathematical system theory. He contributed to the theory of periodic and hybrid systems, delay-differential systems, model reduction for nonlinear systems, control with communication constraints, locomotion and optimal control. He served on several conference IPC's and is a member of the IFAC Committee on Linear Systems, a Fellow of the IEEE (2012), a member of the Royal Flemish Academy of Belgium for Science and the Arts (elected 2012), and a Distinguished Professor of ECE, Georgia Tech (2014). He was a plenary speaker at the 2nd IFAC Workshop on Linear Time Delay Systems in Grenoble (2000) and the 13-th IFAC Workshop on Time Delay Systems in Istanbul (2016). Presently he holds the Giovanni Prodi Chair at the Mathematics Institute of the Julius Maximilians Universität Würzburg, Germany.

VII. Invited Industrial Talk and Panel Discussion

Invited Talk "How to Effectively Evaluate the Stability of Large Power Systems, Including Short-Term Stochastic Behaviors"

Patrick Panciatici (Scientific Advisor at RTE (French TSO))

Dime and Time: Wednesday, July 20, 11:00-11:30 (Paris Time)

Abstract

Stability assessment of large power systems is usually performed using time domain simulation. Numerical schemes have been developed in the last decades, based on hybrid DAE solvers (with switches) using a predictor-corrector approach (typically IDA from sundials: IDA | Computing (llnl.gov)). They are quite efficient in the deterministic case. But in fact, there are always short-term stochastic effects in large power systems that are neglected in this type of stability assessment, Unfortunately, the deterministic approach does not verify that these stochastic effects do not excite instabilities and our classical predictor-corrector approach seems incompatible with noise addition. In the presentation, we will present an example and a formulation of the problem.

Biography

Patrick Panciatici is a graduate of Supélec. He joined EDF R&D in 1985 and then RTE in 2003 when he participated in the creation of an internal R&D department at RTE. He has more than 35 years of experience in power systems: modeling, simulation, control and optimization. Currently, as a scientific advisor, he inspires and coordinates RTE's long-term research on the "system" dimension. He interacts with a large network of international experts and with academic teams worldwide on these topics. He is a member of CIGRE, Fellow of IEEE, RTE representative in PSERC and Bits & Watts.

Panel Discussion "The Interplay between Industrial Challenges and Academic Research"

Dime and Time: Wednesday, July 20, 11:30-11:20 (Paris Time)

Panelists:

- Saber Elaydi (Trinity University, USA)
- Alessio Iovine (CNRS - L2S, France)
- Ernesto Kofman (CIFASIS - CONICET, Agentina)
- Houria Siguerdidjane (CentraleSupélec, France)

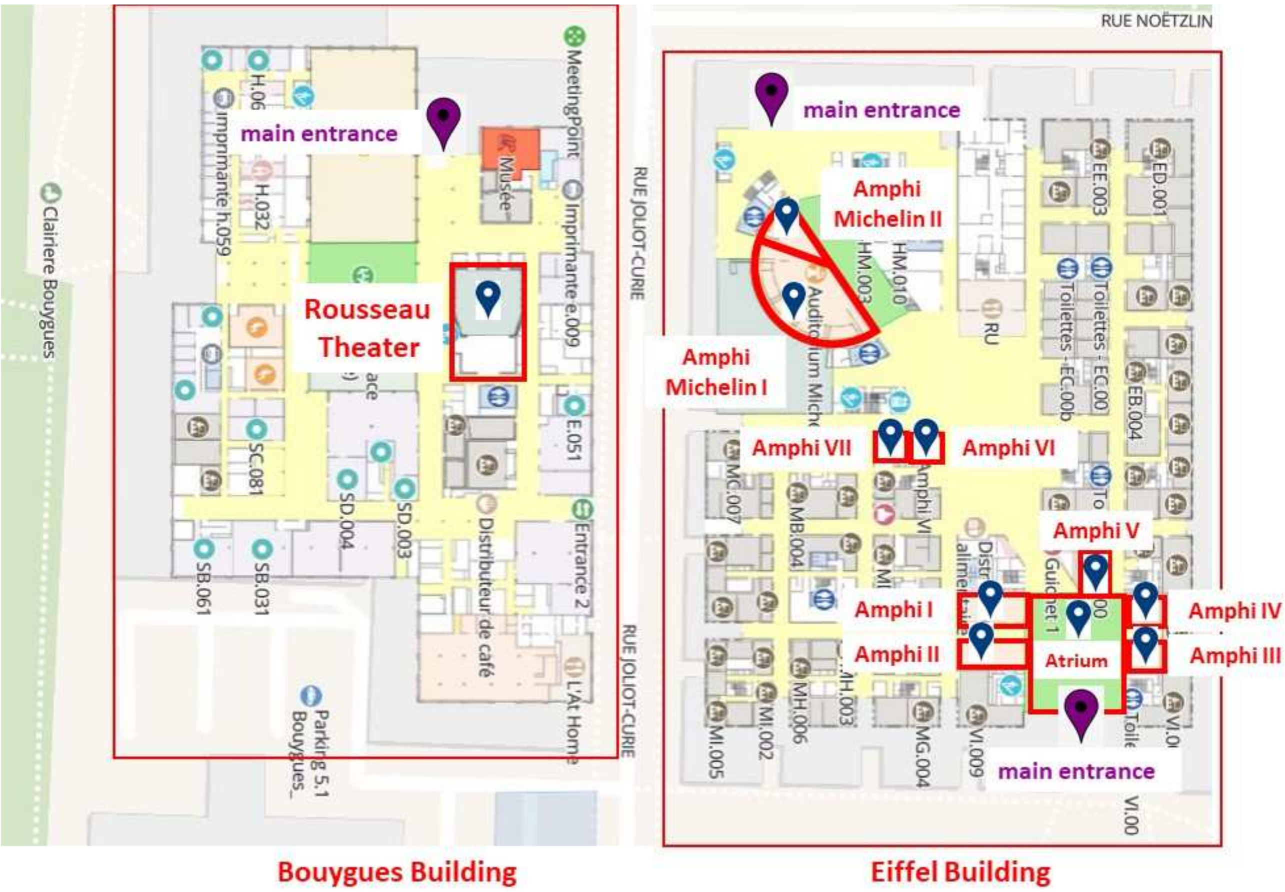
VIII. Local Information

Location

CentraleSupélec is a French graduate engineering school of Paris-Saclay University, located 30km to the south of Paris.

ICDEA will be held in:

- **Eiffel Building** of CentraleSupélec, located at 8-10 rue Joliot-Curie, 91190 Gif-sur-Yvette (GPS Map 48°42'36.1"N 2°10'02.9"E)
- **Bouygues Building** of CentraleSupélec, located at 9, 9 Bis, 9 Ter rue Joliot-Curie, Gif-sur-Yvette (GPS Map 48°42'33.8"N 2°10'00.4"E).



Map of the Eiffel and Bouygues Buildings

Social Events

- **Opening Gala**: Monday, July 18 at 6 pm, Clairière CentraleSupélec

A series of visits and social events will be organized at:

- **Synchrotron SOLEIL**: Tuesday, July 19, starting from 4 pm. Further information will be provided soon.

- **Château de Versailles**: Wednesday, July 20, starting from 2:15 pm.

Schedule

- **2:15 pm** Meeting at the bus at the conference place
- **3:00 pm** Arrival at Versailles, group photo and short walk around the castle
- **4:15 pm**: Meeting point with the guide(s) in front of the gate of the chapel of Palace of Versailles

- **4:30 pm**: Guided Tour of the Grands Apartments of the Versailles Palace (1h30)
  A guide is assigned to each group of 30 people maximum, including the accompanying persons. Subject to changing health conditions. Respect for social distancing and barrier gestures. Earphones are mandatory.
  Self-guided tour of the French gardens with the musical gardens
  End of the visit - Walk to the restaurant
- **7:00 pm**: Dinner at the hotel Le Louis M Gallery by Sofitel, in Versailles

• **Banquet -- Dinner cruise on the River Seine**: Thursday, July 21, starting from 6:15 pm.

Discover or rediscover Paris during a cruise of about 2h30 while enjoying a dinner. The cruise allows you to discover the Parisian monuments along the Seine from the Ile aux Cygnes to Bercy.

Schedule

- **6:15 pm**: Convocation of the participants: CentraleSupélec campus, in front of the Eiffel Building, 8-10 rue Joliot-Curie 91190 Gif-sur-Yvette,
- **6:30 pm**: Departure from Gif-sur-Yvette to Paris, by bus (about 1h30),
- **7:45-8:00 pm**: Arrival at the pier of the Paris cruises,
- **8:30 pm**: Departure of the cruise. Enjoy the charm of the illuminated monuments of Paris and savor a sumptuous dinner prepared on board by the Chef.

  Taste French gastronomy: The Chef proposes a traditional French cuisine, elegant and refined, prepared daily on board with fresh and seasonal products. It favors taste, simplicity and respect for the original flavors of the ingredients. A mastered culinary technique and a meticulous presentation of the dishes sublimate each plate served on board each plate served on board our cruises.

IX. Program at a Glance

ICDEA 2022 Technical Program Monday July 18, 2022			
09:30-10:00 Amphi Michelin I, Eiffel Building Opening Ceremony			
10:00-11:00 Amphi Michelin I, Eiffel Building Azmy S. Ackleh -- "Application of Discrete-Time Stage Structured Models to Understand the Dynamics of a Tick Population and Associated Pathogens"			
11:00-12:00 Amphi III, Eiffel Building Difference Equations and their Applications in Biology	11:00-12:00 Amphi IV, Eiffel Building Special Functions and Orthogonal Polynomials	11:00-12:00 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications	11:00-12:00 Amphi VI, Eiffel Building Difference-Differential Equations

13:00-14:00 Amphi Michelin I, Eiffel Building Gian Italo Bischi – "Discrete Dynamic Models in Social Sciences: Strategic Interaction, Rationality, Evolution."		
14:00-15:30 Amphi III, Eiffel Building Difference Equations and their Applications in Biology	14:00-15:30 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	14:00-15:30 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications

16:00-17:00 Amphi Michelin I, Eiffel Building Dorothée Normand-Cyrot – "Nonlinear Discrete-Time Dynamics in Port-Hamiltonian Form."		
17:00-18:00 Amphi III, Eiffel Building Difference Equations and their Applications in Biology	17:00-18:00 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	17:00-18:00 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications

ICDEA 2022 Technical Program Tuesday July 19, 2022				
08:30-10:30 Amphi III, Eiffel Building Difference Equations and their Applications in Biology	08:30-10:30 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	08:30-10:30 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications	08:30-10:30 Amphi VI, Eiffel Building Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications	08:30-10:30 Amphi VII, Eiffel Building Chaos Theory

11:00-12:00 Amphi Michelin I, Eiffel Building Franco Blanchini – “Mathematics of Mechanisms”

13:00-14:00 Rousseau Theater, Bouygues Building JDEA best paper 2022 talk – "From Elliptic Curves to Bifurcation Theory: Differentiable Versus Topological Conjugacy" by Paul Glendinning and Sasha Glendinning
14:00-15:00 Rousseau Theater, Bouygues Building ISDE General meeting
15:00-16:00 Rousseau Theater, Bouygues Building Suzanne Lenhart – "Two Applications of Discrete Population Models with Hybrid Features”

ICDEA 2022 Technical Program Wednesday July 20, 2022				
08:30-09:30 Amphi III, Eiffel Building Stochastic and Non-Autonomous Difference Systems	08:30-09:30 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	08:30-09:30 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications	08:30-09:30 Amphi VI, Eiffel Building Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications	08:30-09:30 Amphi VII, Eiffel Building Boundary Value Problems

10:00-11:00 Amphi Michelin I, Eiffel Building Piotr Oprocha – "On Typical Properties of Lebesgue Measure Preserving Maps in Dimension One”
11:00-12:00 Amphi Michelin I, Eiffel Building Invited Industrial Talk – "How to Effectively Evaluate the Stability of Large Power Systems, Including Short-Term Stochastic Behaviors" by Patrick Panciatici followed by a Panel Discussion: "The Interplay between Industrial Challenges and Academic Research"
12:00-13:00 Amphi Michelin I, Eiffel Building Carsten Schneider – “Symbolic Summation, Linear Difference Equations and Challenging Applications”

ICDEA 2022 Technical Program Thursday July 20, 2022				
08:30-09:30 Amphi III, Eiffel Building Stochastic and Non-Autonomous Difference Systems	08:30-09:30 Amphi IV, Eiffel Building Complex Dynamics	08:30-09:30 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications	08:30-09:30 Amphi VI, Eiffel Building Difference-Differential Equations	08:30-09:30 Amphi VII, Eiffel Building Control Theory
10:00-11:00 Amphi Michelin I, Eiffel Building Hinke Osinga – "Phase resetting as a two-point boundary value problem"				
11:00-12:00 Amphi III, Eiffel Building Stochastic and Non-Autonomous Difference Systems	11:00-12:00 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	11:00-12:00 Amphi V, Eiffel Building Symmetries and Integrable Systems	11:00-12:00 Amphi VI, Eiffel Building Difference-Differential Equations	11:00-12:00 Amphi VII, Eiffel Building New Trends in Dynamic Geometry
13:00-14:00 Amphi Michelin I Eckehard Schöll – "Synchronization Patterns and Chimera States in Networks of Coupled Maps"				
14:00-15:30 Amphi III, Eiffel Building Stochastic and Non-Autonomous Difference Systems	14:00-15:30 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	14:00-15:30 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications	14:00-15:30 Amphi VI, Eiffel Building Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications	14:00-15:30 Amphi VII, Eiffel Building New Trends in Dynamic Geometry
15:30-16:00 Atrium, Eiffel Building Poster Session				
16:00-17:00 Amphi III, Eiffel Building Stochastic and Non-Autonomous Difference Systems	16:00-17:00 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	16:00-17:00 Amphi V, Eiffel Building Iteration Theory	16:00-17:00 Amphi VI, Eiffel Building Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications	16:00-17:00 Amphi VII, Eiffel Building Numerical Analysis
17:00-18:00 Amphi Michelin I, Eiffel Building Silviu-Iulian Niculescu – "Delays, Interconnections and Control. A Guided Tour"				

ICDEA 2022 Technical Program Friday July 21, 2022				
08:30-09:30 Amphi III, Eiffel Building Linear Difference Equations	08:30-09:30 Amphi IV, Eiffel Building Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems	08:30-09:30 Amphi V, Eiffel Building Other Topics in Difference Equations	08:30-09:30 Amphi VI, Eiffel Building Difference-Differential Equations	08:30-09:30 Amphi VII, Eiffel Building Stability

10:00-11:00 Amphi Michelin I, Eiffel Building Erik I. Verriest – “Stability and Realization of Difference Equations over $\mathbb{Z}$ and $\mathbb{R}$”				
11:00-12:30 Amphi III, Eiffel Building Applications	11:00-12:30 Amphi IV, Eiffel Building Dynamic Equations on Time-Scale	11:00-12:30 Amphi V, Eiffel Building Bifurcation in Invertible and Noninvertible Maps: Theory and Applications	11:00-12:30 Amphi VI, Eiffel Building Difference-Differential Equations	11:00-12:30 Amphi VII, Eiffel Building Topological and Combinatorial Dynamics
12:30-13:00 Amphi Michelin I, Eiffel Building Closing Ceremony				

IX. Content List

Monday July 18

Plenary talk: Azmy S. Ackleh

Application of Discrete-Time Stage Structured Models to Understand the Dynamics of a Tick Population and Associated Pathogens

Monday, July 18th 2022
10:00 - 11:00
Amphi Michelin I, Eiffel Building
Chair: René Lozi

Special/Regular sessions

Special session: Difference Equations and their Applications in Biology
Monday, July 18th 2022
11:00 - 12:00
Amphi III, Eiffel Building
Chair: Amy Veprauskas

Global dynamics and stability of discrete dynamical systems/difference equations and application to evolutionary population models
11:00 - 11:30
Speaker: Saber Elaydi
Abstract page: 81

Discrete-Time Models and a SARS CoV-2 Mystery: Sub-Saharan Africa's Low SARS CoV-2 Disease Burden
11:30 - 12:00
Speaker: Nourridine Siewe
Abstract page: 53, 54

Regular session: Special Functions and Orthogonal Polynomials
Monday, July 18th 2022
11:00 - 12:00
Amphi IV, Eiffel Building
Chair: Eylem Öztürk

The limit as $p \longrightarrow \infty$ for the p-Laplacian Equation with Dynamical Boundary Conditions
11:00 - 11:30
Speaker: Eylem Öztürk
Abstract page: 167

On sections of the generating series of the solution to the multidimensional difference equation
11:30 - 12:00
Speaker: Alexander Lyapin
Abstract page: 127, 128

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Monday, July 18th 2022
11:00 - 12:00
Amphi V, Eiffel Building
Chair: Iryna Sushko

Environmental sustainability, nonlinear dynamics and chaos reloaded: 0 matters!
11:00 - 11:30
Speaker: Andrea Caravaggio
Abstract page: 68, 69

Endogenous preferences in a dynamic Cournot duopoly
11:30 - 12:00
Speaker: Mauro Sodini
Abstract page: 66, 67

Regular session: Difference-Differential Equations
Monday, July 18th 2022
11:00 - 12:00
Amphi VI, Eiffel Building
Chair: Andrzej Nowakowski

Discrete impulses in difference-differential equations with generalized proportional Caputo fractional derivative and stability
11:00 - 11:30
Speaker: Snezhana Hristova
Abstract page: 50, 51

Difference equations and optimal control in machine learning
11:30 - 12:00
Speaker: Andrzej Nowakowski
Abstract page: 49

Plenary talk: Gian Italo Bischi

Discrete Dynamic Models in Social Sciences: Strategic Interaction, Rationality, Evolution.

Monday, July 18th 2022
13:00 - 14:00
Amphi Michelin I, Eiffel Building
Chair: Laura Gardini

Special sessions

Special session: Difference Equations and their Applications in Biology
Monday, July 18th 2022
14:00 - 15:30
Amphi III, Eiffel Building
Chair: Azmy Ackleh

A quadratic Lyapunov function for the planar Ricker map
14:00 - 14:30
Speaker: Stephen Baigent
Abstract page: 11, 12

Periodic maps of mixed monotonicity
14:30 - 15:00
Speaker: Ziyad Al Sharawi
Abstract page: 138

The evolutionary stability of partial migration with allee effects
15:00 - 15:30
Speaker: Anushaya Mohapatra
Abstract page: 166

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Monday, July 18th 2022
14:00 - 15:30
Amphi IV, Eiffel Building
Chair: Davor Dragičević

Dynamics of Iteration Operator
14:00 - 14:30
Speaker: Weinian Zhang
Abstract page: 59

Input-Output Criteria for Stability of Discrete Dynamical Systems and Applications
14:30 - 15:00
Speaker: Adina Luminita Sasu
Abstract page: 93, 94

On the linearization of infinite-dimensional random dynamical systems
15:00 - 15:30
Speaker: Davor Dragičević
Abstract page: 131

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Monday, July 18th 2022
14:00 - 15:30
Amphi V, Eiffel Building
Chair: Eddy Kwessi

A Dynamic Cournot Duopoly with Differentiated Goods and Emission Charges on Output: Linear and Nonlinear Formulations
14:00 - 14:30
Speaker: Marina Pireddu
Abstract page: 6, 7

Big or small? A new economic geography model with an endogenous switch in the market structure
14:30 - 15:00
Speaker: Pasquale Commendatore
Abstract page: 34

Allee plasticity rule in unsupervised learning environment
15:00 - 15:30
Speaker: Eddy Kwessi
Abstract page: 20

Plenary talk: Dorothée Normand-Cyrot

Nonlinear Discrete-Time Dynamics in Port-Hamiltonian Form

Monday, July 18th 2022
16:00 - 17:00
Amphi Michelin I, Eiffel Building
Chair: Elena Braverman

Special sessions

Special session: Difference Equations and their Applications in Biology
Monday, July 18th 2022
17:00 - 18:00
Amphi III, Eiffel Building
Chair: Azmy Ackleh

Method to Derive Discrete Population Models and their Continuous Counterparts
17:00 - 17:30
Speaker: Sabrina Streipert
Abstract page: 103

The Fundamental Theorem of Demography
17:30 - 18:00
Speaker: Jim Cushing
Abstract page: 168

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Monday, July 18th 2022
17:00 - 18:00
Amphi IV, Eiffel Building
Chair: Adina Luminita Sasu

A note on the differentiability of discrete Palmer's linearization
17:00 - 17:30
Speaker: Alvaro Castañeda
Abstract page: 14, 15

Dynamics of a delayed discrete size--structured chemostat with variable nutrient supply
17:30 - 18:00
Speaker: Gonzalo Robledo
Abstract page: 62, 63

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Monday, July 18th 2022
17:00 - 18:00
Amphi V, Eiffel Building
Chair: Davide Radi

Sentiment-driven financial market dynamics: Mathematical insights from a 2D nonsmooth map
17:00 - 17:30
Speaker: Iryna Sushko
Abstract page: 151, 152

Insights on the Dynamics of a Piecewise-Linear Model for the Exchange Rate
17:30 - 18:00
Speaker: Davide Radi
Abstract page: 95, 96

Tuesday July 19

Special/Regular sessions

Special session: Difference Equations and their Applications in Biology
Tuesday, July 19th 2022
08:30 - 10:30
Amphi III, Eiffel Building
Chair: Azmy Ackleh

Dispersal-Driven Coexistence in a Multi-Patch Competition Model for Zebra and Quagga Mussels
08:30 - 09:00
Speaker: Paul Salceanu
Abstract page: 55, 56

Final Size of network SIR model and Katz-Bonacich centrality
09:00 - 09:30
Speaker: Andrés Angel
Abstract page: 76

Boundedness and global attractivity to three-species cyclic prey-predator of Volterra type difference equations
09:30 - 10:00
Speaker: Yoshihiro Hamaya
Abstract page: 41

Stage-structure in interacting species models
10:00 - 10:30
Speaker: Amy Veprauskas
Abstract page: 163

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Tuesday, July 19th 2022
08:30 - 10:30
Amphi IV, Eiffel Building
Chair: Davor Dragičević

Lyapunov exponents for transfer operator cocycles of metastable maps: a quarantine approach
08:30 - 09:00
Speaker: Cecilia Gonzalez-Tokman
Abstract page: 100

Angular Values of Linear Discrete Time Dynamical Systems I
09:00 - 09:30
Speaker: Thorsten Hüls
Abstract page: 29

Angular Values of Linear Discrete Time Dynamical Systems II
09:30 - 10:00
Speaker: Wolf-Juergen Beyn
Abstract page: 30

Quenched linear response for expanding on average cocycles
10:00 - 10:30
Speaker: Julien Sedro
Abstract page: 139

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Tuesday, July 19th 2022
08:30 - 10:30
Amphi V, Eiffel Building
Chair: Anastasiia Panchuk

Bistability-affected Period Adding in a Power Inverter with Hysteresis Control
08:30 - 09:00
Speaker: Zhanybai T. Zhusubaliyev
Abstract page: 37, 38

Period incrementing and Milnor attractors for non autonomous families of flat top tent maps
09:00 - 09:30
Speaker: Luis Silva
Abstract page: 136

Dynamics of Some Discontinuous Discrete Population Models
09:30 - 10:00
Speaker: Vlajko Kocic
Abstract page: 60, 61

Exterior, interior and expansion-like border collisions for chaotic attractors in 1D discontinuous maps
10:00 - 10:30
Speaker: Anastasiia Panchuk
Abstract page: 75

Special session: Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications
Tuesday, July 19th 2022
08:30 - 10:30
Amphi VI, Eiffel Building
Chair: Federico Zullo

A Non-resonant Nabla Fractional Boundary Value Problem
08:30 - 09:00
Speaker: Jaganmohan Jonnalagadda
Abstract page: 9, 10

Abstract fully nonlinear equations with fractional time derivative
09:00 - 09:30
Speaker: Davide Guidetti
Abstract page: 18

Viscoelasticity and magneto-viscoelasticity: an overview on recent results
09:30 - 10:00
Speaker: Sandra Carillo
Abstract page: 176

Degeneration and Homogenization of Fokker-Planck diffusion
10:00 - 10:30
Speaker: Daniele Andreucci
Abstract page: 46, 47

Regular session: Chaos Theory
Tuesday, July 19th 2022
08:30 - 10:30
Amphi VII, Eiffel Building
Chair: Sergey Kryzhevich

Grazing bifurcation and non-uniform hyperbolicity
08:30 - 09:00
Speaker: Sergey Kryzhevich
Abstract page: 84, 85

Other levels of chaos for Turing Machine's systems
09:00 - 09:30
Speaker: Mauricio Diaz
Abstract page: 135

On discrete homoclinic attractors of three-dimensional diffeomorphisms
09:30 - 10:00
Speaker: Alexandr Gonchenko
Abstract page: 120, 121

On bifurcations of Lorenz and Rovella attractors in the Lyubimov-Zaks model
10:00 - 10:30
Speaker: Alexey Kazakov
Abstract page: 119

Plenary talk: Franco Blanchini

Mathematics of Mechanisms

Tuesday, July 19th 2022
11:00 - 12:00
Amphi Michelin I, Eiffel Building
Chair: Sorin Olaru

JDEA best paper 2022 talk: Paul Glendinning and Sacha Glendinning

From Elliptic Curves to Bifurcation Theory: Differentiable Versus Topological Conjugacy
Tuesday, July 19th 2022
13:00 - 14:00
Rousseau Theater, Bouygues Building
Chair: Saber Elaydi

ISDE General meeting

Tuesday, July 19th 2022
14:00 - 15:00
Rousseau Theater, Bouygues Building
Chair: Saber Elaydi

Plenary talk: Suzanne Lenhart

Two Applications of Discrete Population Models with Hybrid Features
Tuesday, July 19th 2022
15:00 - 16:00
Rousseau Theater, Bouygues Building
Chair: Jim Cushing

Wednesday July 20

Special/Regular sessions

Special session: Stochastic and Non-Autonomous Difference Systems
Wednesday, July 20th 2022
08:30 - 09:30
Amphi III, Eiffel Building
Chair: Elena Braverman

Stochastic fractional 2D-Stokes model with delay
08:30 - 09:00
Speaker: Tomás Caraballo
Abstract page: 164

Analysis of time fractional delay 2D-Navier-Stokes equations driven by colored noise
09:00 - 09:30
Speaker: Jiaohui Xu
Abstract page: 26

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Wednesday, July 20th 2022
08:30 - 09:30
Amphi IV, Eiffel Building
Chair: Davor Dragičević

Nonuniform Exponential Dichotomy and Admissibility
08:30 - 09:00
Speaker: Linfeng Zhou
Abstract page: 110

Recent Progresses on C^1 Linearization of Hyperbolic Dynamical Systems
09:00 - 09:30
Speaker: Wenmeng Zhang
Abstract page: 145

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Wednesday, July 20th 2022
08:30 - 09:30
Amphi V, Eiffel Building
Chair: Laura Gardini

A discontinuous model of exchange rate dynamics
08:30 - 09:00
Speaker: Fabio Tramontana
Abstract page: 13

Appearance of closed invariant curves in a piecewise Cournot model with advertising
09:00 - 09:30
Speaker: Nicolò Pecora
Abstract page: 31

Special session: Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications
Wednesday, July 20th 2022
08:30 - 09:30
Amphi VI, Eiffel Building
Chair: Sandra Carillo

p-Homogenization of Fractal Membranes
08:30 - 09:00
Speaker: Simone Creo
Abstract page: 179

On non autonomous Venttsel problems in fractal domains
09:00 - 09:30
Speaker: Maria Rosaria Lancia
Abstract page: 126

Regular session: Boundary Value Problems
Wednesday, July 20th 2022
08:30 - 09:30
Amphi VII, Eiffel Building
Chair: Nick Fewster-Young

Reduction of Systems of Linear 2-D Difference Equations to a Single Equation
8:30 - 9:00
Speaker: Mohamed Salah Boudellioua
Abstract page: 148, 149

Discrete Fractional Boundary Value Problems
09:00 - 09:30
Speaker: Nick Fewster-Young
Abstract page: 74

Plenary talk: Piotr Oprocha

On Typical Properties of Lebesgue Measure Preserving Maps in Dimension One
Wednesday, July 20th 2022
10:00 - 11:00
Amphi Michelin I, Eiffel Building
Chair: Stephen Baigent

Invited Industrial Talk and Panel Discussion

Invited Industrial Talk *"How to Effectively Evaluate the Stability of Large Power Systems, Including Short-Term Stochastic Behaviors"* by Patrick Panciatici followed by a Panel Discussion *"The Interplay between Industrial Challenges and Academic Research"*
Wednesday, July 20th 2022
11:00 - 12:00
Amphi Michelin I, Eiffel Building
Chair: Alessio Iovine

Plenary talk: Carsten Schneider

Symbolic Summation, Linear Difference Equations and Challenging Applications
Wednesday, July 20th 2022
12:00 - 13:00
Amphi Michelin I, Eiffel Building
Chair: Antoine Girard

Thursday July 21

Special/Regular sessions

Special session: Stochastic and Non-Autonomous Difference Systems
Thursday, July 21st 2022
08:30 - 09:30
Amphi III, Eiffel Building
Chair: Cónall Kelly

> *Stable and Historic Behavior in Replicator Equations*
> **08:30 - 09:00**
> Speaker: Mansur Saburov
> Abstract page: 161, 162

> *On Target-Oriented Control of Henon and Lozi maps*
> **09:00 - 09:30**
> Speaker: Elena Braverman
> Abstract page: 118

Regular session: Complex Dynamics
Thursday, July 21st 2022
08:30 - 09:30
Amphi IV, Eiffel Building
Chair: Mariusz Bialecki

> *Discrete probabilistic aggregative dynamics related to integer sequences*
> **08:30 - 09:00**
> Speaker: Mariusz Bialecki
> Abstract page: 52

> *Analytic normalization of strongly hyperbolic (complex) Dulac germs*
> **09:00 - 09:30**
> Speaker: Dino Peran
> Abstract page: 27, 28

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Thursday, July 21st 2022
08:30 - 09:30
Amphi V, Eiffel Building
Chair: Wirot Tikjha

> *Complex Dynamical Behaviors in a Bertrand Game with Service Factor and Differentiated Products*
> **08:30 - 09:00**
> Speaker: Wei Zhou
> Abstract page: 43

> *Further properties in discrete time Lorenz model*
> **09:00 - 09:30**
> Speaker: Wirot Tikjha
> Abstract page: 80

Regular session: Difference-Differential Equations
Thursday, July 21st 2022
08:30 - 09:30
Amphi VI, Eiffel Building
Chair: Chris Lynd

Rational-Linear Anticompetitive Systems of Difference Equations
08:30 - 09:00
Speaker: Chris Lynd
Abstract page: 142

Global Dynamics in Discrete Versions of Differential Delay Models
09:00 - 09:30
Speaker: Anatoli Ivanov
Abstract page: 82

Regular session: Control Theory
Thursday, July 21st 2022
08:30 - 09:30
Amphi VII, Eiffel Building
Chair: Ewa Girejko

Hautus-Yamamoto criteria for approximate and exact controllability of difference delay equations
08:30 - 09:00
Speaker: Sebastien Fueyo
Abstract page: 86, 87

On impact of disturbance in the deployment problem of multi-agent system
09:00 - 09:30
Speaker: Katarzyna Topolewicz
Abstract page: 123, 124

Plenary talk: Hinke Osinga

Phase Resetting as a Two-Point Boundary Value Problem
Thursday, July 21st 2022
10:00 - 11:00
Amphi Michelin I, Eiffel Building
Chair: John Appleby

Special/Regular sessions

Special session: Stochastic and Non-Autonomous Difference Systems
Thursday, July 21st 2022
11:00 - 12:00
Amphi III, Eiffel Building
Chair: Elena Braverman

Asymptotic characterisation of the mean square of solutions of perturbed linear stochastic difference equations
11:00 - 11:30
Speaker: John Appleby
Abstract page: 32

Real exponential asymptotic behaviour is generic in the mean square of two-dimensional linear stochastic difference equations
11:30 - 12:00
Speaker: Emmet Lawless
Abstract page: 143, 144

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Thursday, July 21st 2022
11:00 - 12:00
Amphi IV, Eiffel Building
Chair: Adina Luminita Sasu

On the relation between growth conditions on some operator means and the behavior of the resolvent for bounded operators on Banach spaces
11:00 - 11:30
Speaker: Laurian Suciu
Abstract page: 132, 133

Theoretical and practical aspects for discrete optimal partitioning problems
11:30 - 12:00
Speaker: Beniamin Bogosel
Abstract page: 174

Regular session: Symmetries and Integrable Systems
Thursday, July 21st 2022
11:00 - 12:00
Amphi V, Eiffel Building
Chair: Adam Doliwa

On integrable partial difference equations in rational approximation/interpolation theory
11:00 - 11:30
Speaker: Adam Doliwa
Abstract page: 125

Darboux and Bäcklund transformations for integrable difference equations
11:30 - 12:00
Speaker: Pavlos Xenitidis
Abstract page: 45

Regular session: Difference-Differential Equations
Thursday, July 21st 2022
11:00 - 12:00
Amphi VI, Eiffel Building
Chair: Manjunath Gandhi

ε-neighborhoods of orbits of time-one maps
11:00 - 11:30
Speaker: Vesna Županović
Abstract page: 70, 71

Dynamical Data to Difference Equations
11:30 - 12:00
Speaker: Manjunath Gandhi
Abstract page: 58

Special session: New Trends in Dynamic Geometry
Thursday, July 21st 2022
11:00 - 12:00
Amphi VII, Eiffel Building
Chair: Suthep Suantai

On the RPI approximations of the mRPI set in the case of zonotopic disturbances
11:00 - 11:30
Speaker: Florin Stoican
Abstract page: 130

Dynamic Geometry Involving Kasner Polygons with Complex Parameter
11:30 - 12:00
Speaker: Ovidiu Bagdasar
Abstract page: 57

Plenary talk: Eckehard Schöll

Synchronization Patterns and Chimera States in Networks of Coupled Maps
Thursday, July 21st 2022
13:00 - 14:00
Amphi Michelin I, Eiffel Building
Chair: Silviu-Iulian Niculescu

Special sessions

Special session: Stochastic and Non-Autonomous Difference Systems
Thursday, July 21st 2022
14:00 - 15:30
Amphi III, Eiffel Building
Chair: Elena Braverman

Adaptivity and splitting for the numerical solution of SDEs with square-root diffusion
14:00 - 14:30
Speaker: Cónall Kelly
Abstract page: 19

Graphon Dynamical Systems: a Law of Large Numbers, Large Deviations, and Applications
14:30 - 15:00
Speaker: Georgi Medvedev
Abstract page: 83

Numerical simulations of fractional variable-order dynamics in consensus modelling
15:00 - 15:30
Speaker: Dorota Mozyrska

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Thursday, July 21st 2022
14:00 - 15:30
Amphi IV, Eiffel Building
Chair: Davor Dragičević

Chaotic Neuron Model with Periodic Cofficients
14:00 - 14:30
Speaker: Inese Bula
Abstract page: 42

Weakly nonwandaring points in dynamics of skew products in high dimensions
14:30 - 15:00
Speaker: Lyudmila Efremova
Abstract page: 177

Stability analysis of two-term fractional-order difference equations
15:00 - 15:30
Speaker: Oana Brandibur
Abstract page: 158

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Thursday, July 21st 2022
14:00 - 15:30
Amphi V, Eiffel Building
Chair: Henrique M. Oliveira

Complex dynamics and bifurcations in a contest model with endogenous preferences
14:00 - 14:30
Speaker: Fausto Cavalli
Abstract page: 44

Bifurcation equations for periodic orbits of implicit discrete dynamical systems
14:30 - 15:00
Speaker: Henrique M. Oliveira
Abstract page: 33

Snap-back repeller and Shilnikov singular chaotic attractor in the simplest neuron model
15:00 - 15:30
Speaker: Nataliya Stankevich
Abstract page: 153, 154

Special session: Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications
Thursday, July 21st 2022
14:00 - 15:30
Amphi VI, Eiffel Building
Chair: Sandra Carillo

Some results about p-cycles of third order
14:00 - 14:30
Speaker: Antonio Linero Bas
Abstract page: 157

On the relationship between Lozi maps and max-type difference equations
14:30 - 15:00
Speaker: Daniel Nieves Roldán
Abstract page: 134

Recurrence Coefficients of Orthogonal Polynomials and Painlevé Equations
15:00 - 15:30
Speaker: Galina Filipuk
Abstract page: 146

Special session: New Trends in Dynamic Geometry
Thursday, July 21st 2022
14:00 - 15:30
Amphi VII, Eiffel Building
Chair: Ovidiu Bagdasar

An Accelerated Convex Optimization Algorithm with Line Search and Applications in Machine Learning
14:00 - 14:30
Speaker: Suthep Suantai
Abstract page: 21

A Centralized Three-Term Conjugate Gradient Method for Variational Inequality Problem over the Common Fixed-Point Constraints
14:30 - 15:00
Speaker: Narin Petrot
Abstract page: 2, 3

Modified Mann-type iterative algorithm for solving fixed point and monotone problems with applications to image restoration problems
15:00 - 15:30
Speaker: Kasamsuk Ungchittrakool
Abstract page: 106, 107

Poster session

Thursday, July 21st 2022
15:30 - 16:00
Atrium, Eiffel Building

A Level-Set Method for Numerical Analysis of Compressible Bubble Motion with Phase Change
Author: Gihun Son
Abstract page: 8

Equilibrium and Stability Analysis of a Model of Immune Competition
Author: Bouchra Aylaj
Abstract page: 72, 73

How predators choose their prey to maximize their utility functions by using switching prey
Authors: Asmaa Idmbarek, Yamna Achik, Hajar Nafia, Imane Agmour, Youssef El Foutayeni
Abstract page: 88, 89

Hyperchaotic Attractors in the Model of Two Coupled Parabola Map
Author: Efrosiniia Karatetskaia
Abstract page: 90, 91

Mathematical study of pollution negative effects of three scenarios: Applicated to Sardina pilchardus, Engraulidae and Phocoena phocoena marine populations
Authors: Fatima Ezzahra Ben Dahou, Imane Agmour, Youssef El Foutayeni
Abstract page: 101, 102

On 1:3 Resonance Under Reversible Perturbations of Conservative Cubic Hénon Maps
Author: Evgeniya Samylina
Abstract page: 114, 115

Special/Regular sessions

Special session: Stochastic and Non-Autonomous Difference Systems
Thursday, July 21st 2022
16:00 - 17:00
Amphi III, Eiffel Building
Chair: Cónall Kelly

Boundedness Character of a Family of First Order Rational Difference Equations with Nonconstant Coefficients
16:00 - 16:30
Speaker: Richard Vernon
Abstract page: 39, 40

An unbounded system of rational difference equations with non-constant coefficients
16:30 - 17:00
Speaker: Zachary Kudlak
Abstract page: 24, 25

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Thursday, July 21st 2022
16:00 - 17:00
Amphi IV, Eiffel Building
Chair: Davor Dragičević

Random-Like properties of Chaotic forcing
16:00 - 16:30
Speaker: Paolo Giulietti
Abstract page: 140

Limit theorems for non-uniformly expanding maps under explicit conditions
16:30 - 17:00
Speaker: Yeor Hafouta
Abstract page: 97

Regular session: Iteration Theory
Thursday, July 21st 2022
16:00 - 17:00
Amphi V, Eiffel Building
Chair: Shamoona Jabeen

Fixed Point Results of Reich Contraction in Fuzzy Metric Spaces Endowed with Graph
16:00 - 16:30
Speaker: Shamoona Jabeen
Abstract page: 77

On extension of fractional iterates of a Brouwer homeomorphism from a parallelizable region
16:30 - 17:00
Speaker: Zbigniew Leśniak
Abstract page: 122

Special session: Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications
Thursday, July 21st 2022
16:00 - 17:00
Amphi VI, Eiffel Building
Chair: Galina Filipuk

The importance of being (discrete) variational
16:00 - 16:30
Speaker: Giorgio Gubbiotti
Abstract page: 169, 170

Recursions and evolutions for zeros of entire functions
16:30 - 17:00
Speaker: Federico Zullo
Abstract page: 147

Regular session: Numerical Analysis
Thursday, July 21st 2022
16:00 - 17:00
Amphi VII, Eiffel Building
Chair: Nicolae Pop

Numerical modeling of dynamic systems describing contact problems with friction in elasticity
16:00 - 16:30
Speaker: Nicolae Pop
Abstract page: 111-113

Bioeconomic model with the presence of tide effects in Moroccan coasts
16:30 - 17:00
Speaker: Nossaiba Baba
Abstract page: 35, 36

Plenary talk: Silviu-Iulian Niculescu

Title Delays, Interconnections and Control. A Guided Tour
Thursday, July 21st 2022
17:00 - 18:00
Amphi Michelin I, Eiffel Building
Chair: Hinke Osinga

Friday July 22

Special/Regular sessions

Regular session: Linear Difference Equations
Friday, July 22nd 2022
08:30 - 09:30
Amphi III, Eiffel Building
Chair: Luis Silva

Periodic and quasi-polynomial solutions of implicit linear difference equations over commutative rings
08:30 - 09:00
Speaker: Anna Goncharuk
Abstract page: 137

Implicit linear difference equation over residue class rings
09:00 - 09:30
Speaker: Mykola Heneralov
Abstract page: 92

Special session: Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems
Friday, July 22nd 2022
08:30 - 09:30
Amphi IV, Eiffel Building
Chair: Adina Luminita Sasu

Stability behaviour of a mathematical model which controls the unemployment and underemployment
08:30 - 09:00
Speaker: Loredana Flavia Vesa
Abstract page: 159, 160

On some dichotomy concepts with growth rates for discrete time systems in Banach spaces
09:00 - 09:30
Speaker: Rovana Boruga
Abstract page: 129

Regular session: Other Topics in Difference Equations
Friday, July 22nd 2022
08:30 - 09:30
Amphi V, Eiffel Building
Chair: Svetlin G. Georgiev

Triple solutions for a Dirichlet boundary value problem involving a perturbed fractional operator
08:30 - 09:00
Speaker: Mohsen Khaleghi Moghadam
Abstract page: 175

ECLSS: Extended Chaotic Map-based Certificateless Signature Scheme using Random Oracle Model
09:00 - 09:30
Speaker: Deepika Kumari
Abstract page: 64, 65

Regular session: Difference-Differential Equations
Friday, July 22nd 2022
08:30 - 09:30
Amphi VI, Eiffel Building
Chair: Snezhana Hristova

Solutions of Generalized Difference Equations and its Applications
08:30 - 09:00
Speaker: Chandrasekar Vadivel
Abstract page: 155, 156

Monotonicity result for nabla fractional h-difference operators
09:00 - 09:30
Speaker: Xiang Liu
Abstract page: 165

Regular session: Stability
Friday, July 22nd 2022
08:30 - 09:30
Amphi VII, Eiffel Building
Chair: Karima Mokni

Lyapunov Functions for Delay Reaction Diffusion Systems
08:30 - 09:00
Speaker: Fatiha Najm
Abstract page: 99

A Discrete Evolutionary Beverthon-Holt model with Allee effect
09:00 - 09:30
Speaker: Karima Mokni
Abstract page: 4, 5

Plenary talk: Erik I. Verriest

Stability and Realization of Difference Equations over $\mathbb{Z}$ and $\mathbb{R}$

Friday, July 22nd 2022
10:00 - 11:00
Amphi Michelin I, Eiffel Building
Chair: Adina Luminita Sasu

Special/Regular sessions

Regular session: Applications
Friday, July 22nd 2022
11:00 - 12:30
Amphi III, Eiffel Building
Chair: Nicolae Pop

The numerical solution of the free-boundary cell motility problem
11:00 – 11:30
Speaker: Vitaly Chernik
Abstract page: 171, 172

Model of evolution of the transmission of Chagas disease
11:30 – 12:00
Speaker: Nabahats Dib-Baghdadli
Abstract page: 104, 105

Regular session: Dynamic Equations on Time-Scale
Friday, July 22nd 2022
11:00 - 12:30
Amphi IV, Eiffel Building
Chair: Vasyl Martsenyuk

On Qualitative Research of Ring Lattice of Neurons with Delayed Coupling on Time Scale
11:00 – 11:30
Speaker: Vasyl Martsenyuk
Abstract page: 116, 117

Well-Posedness of Nonlinear Integro-Dynamic Equations on Time Scales
11:30 – 12:00
Speaker: Sanket Tikare
Abstract page: 178

Linear Dynamic-Algebraic Equations on Time Scales
12:00 – 12:30
Speaker: Svetlin G. Georgiev
Abstract page: 98

Special session: Bifurcation in Invertible and Noninvertible Maps: Theory and Applications
Friday, July 22nd 2022
11:00 - 12:30
Amphi V, Eiffel Building
Chair: Xavier Jarque

Multistability in a stochastic consumption model
11:00 – 11:30
Speaker: Tatyana Perevalova
Abstract page: 108, 109

A Bifurcation analysis of the Microscopic Markov Chain Approach to contact-based epidemic spreading
11:30 – 12:00
Speaker: Antonio Garijo Real
Abstract page: 1

A toy model for the dynamics of the secant map near a critical 3-cycle
12:00 – 12:30
Speaker: Xavier Jarque
Abstract page: 16, 17

Regular session: Difference-Differential Equations
Friday, July 22nd 2022
11:00 - 12:30
Amphi VI, Eiffel Building
Chair: Miloud Assal

Detection of the first nonzero Lyapunov quantity in degenerate slow-fast Hopf bifurcations from fractality of planar contact points
11:00 – 11:30
Speaker: Renato Huzak
Abstract page: 48

An Algebraic Approach to Solve a non Homogeneous Dynamic Cauchy-Euler Equation of High Order
11:30 – 12:00
Speaker: Miloud Assal
Abstract page: 22

An efficient numerical algorithm for pantograph delay differential equations
12:00 – 12:30
Speaker: Giriraj Methi
Abstract page: 23

Regular session: Topological and Combinatorial Dynamics
Friday, July 22nd 2022
11:00 - 12:30
Amphi VII, Eiffel Building
Chair: Michal Misiurewicz

The real teapot
11:00 – 11:30
Speaker: Michal Misiurewicz
Abstract page: 173

Rate of convergence in the disjunctive chaos game
11:30 – 12:00
Speaker: Krzysztof Leśniak
Abstract page: 141

Riordan arrays and Difference equations for restricted lattice paths
12:00 – 12:30
Speaker: Sreelatha Chandragiri
Abstract page: 150

X. Special Sessions

Bifurcation in Invertible and Noninvertible Maps: Theory and Applications

Organizers: **Laura Gardini, Gian Italo Bischi, Iryna Sushko**

A Bifurcation analysis of the Microscopic Markov Chain Approach to contact-based epidemic spreading
Author: Garijo Toni

A discontinuous model of exchange rate dynamics
Authors: Campisi Giovanni, Tramontana Fabio

A Dynamic Cournot Duopoly with Differentiated Goods and Emission Charges on Output: Linear and Nonlinear Formulations
Authors: Naimzada Ahmad, Pireddu Marina

A toy model for the dynamics of the secant map near a critical 3-cycle
Authors: Jarque Xavier, Fontich Ernest, Garijo Antoni

Allee plasticity rule in unsupervised learning environment
Author: Kwessi Eddy

Appearance of closed invariant curves in a piecewise Cournot model with advertising
Authors: Pecora Nicolo, Agliari Anna, Szuz Alina

Bifurcation equations for periodic orbits of implicit dynamical systems
Author: Oliveira Henrique

Big or small? A new economic geography model with an endogenous switch in the market structure
Authors: Commendatore Pasquale, Kubin Ingrid, Sushko Iryna

Bistability-affected Period Adding in a Power Inverter with Hysteresis Control
Author: Zhusubaliyev Zhanybai

Complex Dynamical Behaviors in a Bertrand Game with Service Factor and Differentiated Products
Authors: Zhou Wei, Li Hui

Complex dynamics and bifurcations in a contest model with endogenous preferences
Authors: Cavalli Fausto, Gilli Mario, Naimzada Ahmad

Dynamics of Some Discontinuous Discrete Population Models
Author: Kocic Vlajko

Endogenous preferences in a dynamic Cournot duopoly
Authors: Caravaggio Andrea, Gori Luca, Sodini Mauro

Environmental sustainability, nonlinear dynamics and chaos reloaded: 0 matters!
Authors: Caravaggio Andrea, Sodini Mauro

Exterior, interior and expansion-like border collisions for chaotic attractors in 1D discontinuous maps
Authors: Panchuk Anastasiia, Avrutin Viktor, Sushko Iryna

Further properties in discrete time Lorenz model
Authors: Laura Gardini, Tikjha Wirot

Hyperchaotic Attractors in the Model of Two Coupled Parabola Map
Author: Karatetskaia Efrosiniia

Insights on the Dynamics of a Piecewise-Linear Model for the Exchange Rate
Authors: Gardini Laura, Radi Davide, Schmitt Noemi, Sushko Iryna, Westerhoff Frank

Multistability in a stochastic consumption model
Authors: Jungeilges Jochen, Pavletsov Makar, Perevalova Tatyana

On 1:3 Resonance Under Reversible Perturbations of Conservative Cubic Henon Maps
Author: Samylina Evgeniya

Period incrementing and Milnor attractors for non autonomous families of flat top tent maps
Author: Silva Luis

Sentiment-driven financial market dynamics: Mathematical insights from a 2D nonsmooth map
Authors: Sushko Iryna, Gardini Laura, Radi Davide, Schmitt Noemi, Westerhoff Frank

Snap-back repeller and Shilnikov singular chaotic attractor in the simplest neuron model
Authors: Stankevich Nataliya, Gonchenko Aleksandr

Stochastic and Non-Autonomous Difference Systems

Organizers: **Elena Braverman, Cónall Kelly**

Adaptivity and splitting for the numerical solution of SDEs with square-root diffusion
Author: Cónall Kelly

An unbounded system of rational difference equations with non-constant coefficients **Authors**: Kudlak Zachary, Vernon Patrick

Analysis of time fractional delay 2D-Navier-Stokes equations driven by colored noise
Authors: Xu Jiaohui, Caraballo Tomás

Asymptotic characterisation of the mean square of solutions of perturbed linear stochastic difference equations
Authors: Appleby John, Lawless Emmet

Boundedness Character of a Family of First Order Rational Difference Equations with Nonconstant Coefficients
Author: Vernon Richard

Graphon Dynamical Systems: a Law of Large Numbers, Large Deviations, and Applications
Author: Medvedev Georgi

On Target-Oriented Control of Henon and Lozi maps
Authors: Braverman Elena, Rodkina Alexandra

Real exponential asymptotic behaviour is generic in the mean square of two-dimensional linear stochastic difference equations
Authors: Lawless Emmet, Appleby John

Stable and Historic Behavior in Replicator Equations
Author: Saburov Mansur

Stochastic fractional 2D-Stokes model with delay
Authors: Caraballo Tomás, Xu Jiaohui

Nonlinear Difference and Differential Problems, Transformations, Homogenization Techniques & Applications

Organizers: Sandra Carillo, Galina Filipuk, Federico Zullo

Abstract fully nonlinear equations with fractional time derivative
Author: Guidetti Davide

Degeneration and Homogenization of Fokker--Planck diffusionents
Authors: Andreucci Daniele, Amar Micol, Cirillo Emilio N.M.

On non autonomous Venttsel ' problems in fractal domains
Author: Lancia Maria Rosaria

On the relationship between Lozi maps and max-type difference equations
Author: Nieves Roldán Daniel

p-Homogenization of Fractal Membranes
Author: Simone Creo

Recurrence Coefficients of Orthogonal Polynomials and Painlevé Equations
Author: Filipuk Galina

Recursions and evolutions for zeros of entire functions
Author: Zullo Federico

Some results about p-cycles of third order
Author: Linero Bas Antonio

The importance of being (discrete) variational
Author: Gubbiotti Giorgio

Viscoelasticity and magneto-viscoelasticity: an overview on recent results
Author: Carillo Sandra

Difference Equations and their Applications in Biology

Organizers: Azmy S. Ackleh, Amy Veprauskas

A quadratic Lyapunov function for the planar Ricker map
Authors: Baigent Stephen, Hou Zhanyuan, Elaydi Saber, Balreira Eduardo Cabral, Luis Raphael

Boundedness and global attractivity to three-species cyclic prey-predator of Volterra type difference equations
Author: Hamaya Yoshihiro

Discrete-Time Models and a SARS CoV-2 Mystery: Sub-Saharan Africa's Low SARS CoV-2 Disease Burden
Authors: Siewe Nourridine, Yakubu Abdul-Aziz

Dispersal-Driven Coexistence in a Multi-Patch Competition Model for Zebra and Quagga Mussels
Authors: Salceanu Paul, Huang Qihua, Wang Hao

Final Size of network SIR model and Katz-Bonacich centrality
Author: Angel Andres

Global dynamics and stability of discrete dynamical systems/difference equations and application to evolutionary population models
Author: Elaydi Saber

Method to Derive Discrete Population Models and their Continuous Counterparts
Authors: Streipert Sabrina, Wolkowicz Gail

Periodic maps of mixed monotonicity
Author: Al Sharawi Ziyad

Stage-structure in interacting species models
Authors: Veprauskas Amy, Hossain Istiaq, Jahangir Jenita

The evolutionary stability of partial migration with allee effects
Author: Mohapatra Anushaya

The Fundamental Theorem of Demography
Author: Cushing Jim

Qualitative Behaviour of Nonautonomous Discrete Dynamical Systems

Organizers: Davor Dragičević, Adina Luminita Sasu, Weinian Zhang

A note on the differentiability of discrete Palmer's linearization
Authors: Castañeda Alvaro, Jara Néstor

Angular Values of Linear Discrete Time Dynamical Systems I
Author: Hüls Thorsten

Angular Values of Linear Discrete Time Dynamical Systems II
Author: Beyn Wolf-Juergen

Chaotic Neuron Model with Periodic Coefficients
Author: Bula Inese

Dynamics of a delayed discrete size--structured chemostat with variable nutrient supply
Author: Robledo Gonzalo

Dynamics of Iteration Operator
Author: Zhang Weinian

Input-Output Criteria for Stability of Discrete Dynamical Systems and Applications
Authors: Dragičević Davor, Sasu Adina Luminita, Sasu Bogdan

Limit theorems for non-uniformly expanding maps with explicit conditions
Author: Hafouta Yeor

Lyapunov exponents for transfer operator cocycles of metastable maps: a quarantine approach
Author: Gonzalez Tokman Cecilia

Nonuniform Exponential Dichotomy and Admissibility
Author: Zhou Linfeng

On some dichotomy concepts with growth rates for discrete time systems in Banach spaces
Author: Boruga (Toma) Rovana

On the linearization of infinite-dimensional random dynamical systems
Authors: Davor Dragičević, Backes Lucas

On the relation between growth conditions on some operator means and the behavior of the resolvent for bounded operators on Banach spaces
Authors: Suciu Laurian, Aleman Alexandru

Quenched linear response for expanding on average cocycles
Authors: Sedro Julien, Dragičević Davor, Giulietti Paolo

Random-like properties of chaotic forcing
Author: Giulietti Paolo

Recent Progresses on C^1 Linearization of Hyperbolic Dynamical Systems
Author: Zhang Wenmeng

Stability analysis of two-term fractional-order difference equations
Authors: Brandibur Oana, Kaslik Eva

Stability behaviour of a mathematical model which controls the unemployment and underemployment
Author: Vesa Loredana Flavia

Theoretical and practical aspects for discrete optimal partitioning problems
Author: Bogosel Beniamin

Weakly nonwandaring points in dynamics of skew products in high dimensions
Author: Lyudmila Efremova

New Trends in Dynamic Geometry

Organizers: **Dorin Andrica, Ovidiu Bagdasar**

A Centralized Three-Term Conjugate Gradient Method for Variational Inequality Problem over the Common Fixed-Point Constraints
Authors: Nimana Nimit, Prangprakhon Mootta, Petrot Narin

An Accelerated Convex Optimization Algorithm with Line Searchand Applications in Machine Learning
Author: Suantai Suthep

Dynamic Geometry Involving Kasner Polygons with Complex Parameter
Authors: Bagdasar Ovidiu, Andrica Dorin

Modied Mann-type iterative algorithm for solving fixed point and monotone problems with applications to image restoration problems
Authors: Artsawang Natthaphon, Bagdasar Ovidiu, Ungchittrakool Kasamsuk, Plubtieng Somyot, Baiya Suparat

On the RPI approximations of the mRPI set in the case of zonotopic disturbances
Author: Florin Stoican, Olaru Sorin

XI. Book of Abstracts

A Bifurcation analysis of the Microscopic Markov Chain Approach to contact-based epidemic spreading

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Presentation type: Talk

The problem of modeling the spread of a disease among individuals has been studied in deep over many years. The development of compartmental models, models that divide the individuals among a set of possible states, has given rise to a new collection of techniques that enables, for instance, the analysis of the onset of epidemics, the study of epidemics in structured networks, or the study of the impact of a vaccination campaign. All previous works heavily rely on the mathematical approach to the study of epidemic spreading.

We consider a connected undirected network $\mathcal{N}_n$ made up of n nodes, whose probability connections are represented by the entries $r_{ij} \in [0,1]$ of an $n \times n$ symmetric matrix R. We now define a discrete dynamical system based on the infection process on the network. We introduce two parameters in order to control the epidemic spreading model, the first one is $\beta \in (0,1)$, that controls the probability to transmit the disease to its neighbors, and the second one is $\mu \in (0,1)$, that controls the probability of re-infection. The evolution of this infection process is governed by the iteration of a map $F : \mathbb{R}^n \to \mathbb{R}^n$.

Numerical simulations show that these kind of systems, governed by the map F, converge to an asymptotic distribution

$$\lim_{k \to \infty} F^k(p_1^0, \ldots, p_n^0) = (p_1^\infty, \ldots, p_n^\infty),$$

independently on the initial condition $(p_1^0, \ldots, p_n^0)$. Our goal in to investigate the bifurcation process occurring from an analytic point of view.

A Centralized Three-Term Conjugate Gradient Method for Variational Inequality Problem over the Common Fixed-Point Constraints

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Presentation type: Talk

We suggest a centralized three-term conjugate gradient method for solving the strongly monotone variational inequality problem over the intersection of fixed-point sets of firmly nonexpansive operators. The proposed method allows the independent computation on each firmly nonexpansive operator along with the dynamic weight that depends on each iteration. This strategy aims to speed up the convergence behavior of the algorithm by considering updating control factors to drive each iteration-step. Under some suitable control conditions on corresponding parameters, we show a strong convergence of the iterate to the unique solution of the considered variational inequality problem. We apply the considered model to solve the image classification problem via support vector machine learning method, and show the effectiveness of theoretical results by presenting the interesting numerical experiment observations.

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A Discrete Evolutionary Beverthon-Holt model with Allee effect

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Presentation type: 'Talk'

In this talk, we present a discrete-time evolutionary Beverthon-Holt population model. The aforementioned model is formulated in terms of difference equations and derived following Evolutionary Game Theory, which may be used to model the interplay between population dynamics and evolutionary dynamics. The fixed point's local dynamics are discussed. Furthermore, we show that under certain parametric conditions, the evolutionary model experiences Neimark-Sacker bifurcation in a small neighborhood of the positive fixed point. An unstable fixed point can be stabilized using the OGY methodology and the hybrid method. Analytical conclusions are demonstrated through extensive numerical simulations, which depict the complex dynamics of the developed model.

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A Dynamic Cournot Duopoly with Differentiated Goods and Emission Charges on Output: Linear and Nonlinear Formulations

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Presentation type: Talk, possibly in the Special Session "Bifurcation in Invertible and Noninvertible Maps: Theory and Applications"

We extend the dynamic Cournot duopoly framework with emission charges on output by Mamada and Perrings [2], which encompasses homogeneous products in its original formulation, to the more general case of differentiated goods, in order to highlight the richness in its static and dynamic outcomes. In the model each firm is taxed proportionally to its own emission only and charge functions are quadratic. Moreover, due to an adjustment capacity constraint, firms partially modify their output level toward the best response. Like it happened in [2], the only model steady state coincides with the Nash equilibrium, that will be considered admissible when it guarantees the positivity of the marginal emission charge. We find that the full efficacy of the environmental policy, which applies to an equilibrium that is globally asymptotically stable anytime it is admissible, is achieved in the case of independent goods, as well as with a low interdependence degree between goods in absolute value, independently of being substitutes or complements. On the other hand, when goods are substitutes and their interdependence degree is high, the considered environmental policy is still able to reduce pollution at the equilibrium, but the latter is stable just when the policy intensity degree is high enough. When instead goods are complements and their interdependence degree is high in absolute value, the considered environmental policy produces detrimental effects on the pollution level and the unique equilibrium is always unstable, when admissible. This highlights that, from a static viewpoint, even in the absence of free riding possibilities, the choice of the mechanism to implement has to be carefully pondered, according to the features of the considered economy.
As a second step, since the existing empirical literature [1] suggests that the price and

the exchanged quantity volatility observed in real-world markets may be explained in terms of the endogenous fluctuations generated by the presence of nonlinearities, we replace the linear output adjustment mechanism considered in [2] with a sigmoid mechanism [3], characterized by the presence of two horizontal asymptotes, still focusing on the more general case of differentiated products. In this manner, the model is able to generate interesting, non-divergent dynamic outcomes, despite the linearity of the demand function and of marginal costs. We stress that such modeling choice is also sensible from an economic viewpoint, being suitable to describe the bounded output variations caused by physical, historical and institutional constraints. We start by analyzing in the new setting the stability of the unique steady state, which coincides with the Nash equilibrium, common to the framework in [2], as well as its bifurcations and the role played by the main model parameters, finding that the equilibrium is stable when the two goods are nearly independent, while complex dynamics, following a cascade of period-doubling bifurcations, can arise when the interdependence degree between the two goods is high enough. Moreover, due to the introduction of the sigmoid adjustment mechanism, the equilibrium stability region is reduced with respect to the framework with the linear mechanism. In regard to the environmental policy efficacy, as long as the Nash equilibrium is stable, we find a confirmation of the comparative statics results obtained with the linear adjustment rule in [2]. On the other hand, in view of evaluating the efficacy of the considered environmental policy in the cases in which the Nash equilibrium is not stable, we propose two alternative, dynamic approaches, based either on a comparison of emissions for different levels of charges or on a comparison of emissions along non-stationary trajectories and along the equilibrium path. In particular, when emissions are larger along non-stationary trajectories, we show how to act on the level of the asymptotes of the sigmoid adjustment mechanism in view of decreasing output variations, reaching a complete stabilization of the system and reducing pollution at one time.

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A Level-Set Method for Numerical Analysis of Compressible Bubble Motion with Phase Change

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Presentation type: Poster

Compressible bubble growth and collapse with phase change is of considerable interest in various applications such as thermal inkjet printing, hydrodynamic erosion and medical treatment. Extensive numerical works were conducted for bubble motion with phase change using the front-tracking method [1], the LS method [2], and the VOF method [3]. However, only a few numerical studies were extended to the compressible bubble motion with phase change.

In this work, the LS method is extended to include the effects of bubble compressibility and liquid-vapor phase change by incorporating the semi-implicit pressure correction method as well as the ghost fluid method to efficiently implement the matching conditions of velocity, stress and temperature at the interface. The LS method is applied for numerical analysis of compressible phase-change bubble motion near a wall. The numerical method is further extended to acoustic droplet vaporization (ADV), which can be applied for medical therapy and targeted drug delivery. The ADV, in which volatile droplets immersed in another immiscible liquid are vaporized into bubbles by means of ultrasound, is numerically analyzed introducing two LS functions for the liquid-vapor interface and the liquid-liquid interface.

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A Non-resonant Nabla Fractional Boundary Value Problem

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Presentation type: Talk

We consider a simple boundary value problem for a nabla fractional difference equation. Specifically, we shall consider

$$\begin{cases} \big(\nabla_a^{\nu} u\big)(t) + f(t, u(t)) = 0, \quad t \in \mathbb{N}_{a+2}^{b}, \\ \big(\nabla u\big)(a+1) = 0, \quad \big(\nabla u\big)(b) = 0, \end{cases} \tag{1}$$

where a, $b \in \mathbb{R}$ such that $b - a \in \mathbb{N}_3 = \{3, 4, 5, \cdots\}$, $\mathbb{N}_{a+2}^{b} = \{a+2, a+3, \cdots, b\}$, $f : \mathbb{N}_{a+2}^{b} \times \mathbb{R} \to \mathbb{R}$, $1 < \nu < 2$ and $\nabla_a^{\nu} u$ denotes the ν^{th}-order Riemann–Liouville nabla fractional difference of u based at a.

The boundary value problem (1) is not at resonance because the corresponding homogeneous problem

$$\begin{cases} \big(\nabla_a^{\nu} u\big)(t) = 0, \quad t \in \mathbb{N}_{a+2}^{b}, \\ \big(\nabla u\big)(a+1) = 0, \quad \big(\nabla u\big)(b) = 0, \end{cases} \tag{2}$$

has only the trivial solution.

First, we construct an associated Green's function and obtain some of its properties. Under suitable conditions on the nonlinear part of the nabla fractional difference equation, we deduce a few existence results of the considered nonlinear problem through appropriate fixed point theorems. We also provide a couple of examples to demonstrate the applicability of the established results.

As $\nu \to 2$, the boundary value problem (1) reduces to the following second order discrete boundary value problem.

$$\begin{cases} \big(\nabla^{2} u\big)(t) + f(t, u(t)) = 0, \quad t \in \mathbb{N}_{a+2}^{b}, \\ \big(\nabla u\big)(a+1) = 0, \quad \big(\nabla u\big)(b) = 0, \end{cases} \tag{3}$$

which is at resonance because the corresponding homogeneous problem

$$\begin{cases} \big(\nabla^{2} u\big)(t) = 0, \quad t \in \mathbb{N}_{a+2}^{b}, \\ \big(\nabla u\big)(a+1) = 0, \quad \big(\nabla u\big)(b) = 0, \end{cases} \tag{4}$$

has nontrivial constant solutions. This shows that the memory property of nabla fractional difference operators play an important role in describing the behaviour of solutions of the corresponding nabla fractional difference equations.

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A quadratic Lyapunov function for the planar Ricker map

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Presentation type: Talk

In 1975 May [3] studied the single species Ricker map $x \mapsto xe^{r(1-x)}$, and in particular introduced a quadratic Lyapunov function that demonstrated global asymptotic stability of the positive fixed point when $0 < r < 2$. For the planar Ricker map $(x, y) \mapsto (xe^{r-x-\alpha y}, ye^{s-y-\beta x})$ for $\alpha, \beta, r, s > 0$, a study of global stability of the positive fixed point of the map, when it exists, has proved more challenging. In [7] Smith used monotone systems theory to settled the question for $r, s \leq 1$, but his approach fails for values of r, s exceeding one, as the monotonicity property no longer holds. In [2] the authors studied a symmetric planar Ricker model, using methods for studying non-invertible maps to determine the partition of phase space into regions with constant numbers of preimages and to study bifurcations. More recently, in [1, 4, 5] several authors have explored the global stability of the positive fixed point. For example, in [1] using singularity theory, the authors established conditions on the existence of the attracting invariant manifold by looking at intersections of critical curves and bounds on r, s that also depended on the competition parameters α, β. Recent work in [4, 5], improved the previous results and found optimal bounds of the parameters, but do not cover all parameter cases when $0 < r, s < 2$.

Here we show how to use a simple quadratic Lyapunov function to extend stability analysis to $0 < r, s < 2$. We also use some results of Smith [6] to identify the stable set and unstable manifold that partition the stability basins.

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A discontinuous model of exchange rate dynamics

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Presentation type: 'Talk'

We study the exchange rate dynamics when the market is populated by different kinds of traders. We consider one type of fundamentalists and two types of chartists. Fundamentalists are assumed to react asymmetrically when the exchange rate is above or below its fundamental value, as in Tramontana et al. 2014 [1]. One kind of chartists behave exactly at the opposite of fundamentalists, while the second kind of chartists believe the information in the exchange rate itself carries information about its future dynamics and extrapolate next returns from the current ones, as in Manzan and Westerhoff 2017 [2]. The model is piecewise defined and discontinuous. We study both analytically and numerically the role played by the parameters in the occurrence of bifurcations and we provide an economic interpretation.

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A note on the differentiability of discrete Palmer's linearization

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Presentation type: Talk

The well known Hartman-Grobman theorem [1, Theorem I] is an essential tool in the study of local behavior of autonomous and nonautonomous nonlinear dynamical systems. This theorem establishes a local topological conjugacy between the continuous dynamical system given by a C^1 diffeomophism of the space $\mathbb{R}^d$ and its linearization around an hyperbolic equilibrium, *i.e.*, the dynamics near the equilibrium points are topologically the same.

This work inspired to K.J. Palmer [2] to achieve the first result of global linearization in the nonautonomous framework: The Hartman–Grobman theorem for nonautonomous differential equations. This seminal article considered vector fields whose linear component inherits, in some sense, the hyperbolicity property of the autonomous case, namely uniform exponential dichotomy property, while the nonlinear parts of the vector fields are bounded and small Lipschitzian.

Although the task of finding such a homeomorphism is delicate, to show that it has a class of differentiability is an more interesting challenge in dynamics, since this allows a better understanding of the information that is carried from one system to another.

To the best our knowledge, there are no results about the differentiability of Palmer's linearization before of the work [3], which establishes that, under some technical assumptions, a preserving orientation linearization of class C^2 exists. For a recent generalization and improvement of this result we refer the reader to [4].

Our work follows the results initiated in [3], and is strongly based in the study on discrete framework realized in [5], since we use some of their results and strategies to find a Palmer's homeomorphisms. The main difference between our work and [5], in terms of differentiability, is that we allow the existence of nonempty unstable manifolds for the linear system; furthermore, our analysis does not involves a change from a nonautonomous environment to autonomous one nor the use of spectral gaps in order to obtain C^1-differentiability such as it is employ in [6]. Another interesting fact of our work is that differentiability is reached when in the linear part is consider a wider family of dichotomies.

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A toy model for the dynamics of the secant map near a critical 3-cycle

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Presentation type: Talk

We consider the secant method applied to a polynomial p as a dynamical system acting on the real plane. If the polynomial p has a local extrema at a point α then the secant map exhibits a 3-cycle at the point (α, α). We propose a simple model map T to explain the behavior of S^3 near the point (α, α). In many cases this 3-cycle has a non-empty basin of attraction and their boundary is related to the invariant manifold of some periodic cycle.

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Abstract fully nonlinear equations with fractional time derivative

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Presentation type: Talk

We consider abstract fully nonlinear equations in Banach spaces, with a time derivative of order $\alpha \in (0, 2)$ in the sense of Caputo. We extend to this more general case a classical maximal regularity theorem due to G. Da Prato and P. Grisvard ([1]) for $\alpha = 1$. We apply this to general nonlinear equations by perturbation methods. We study also the behaviour of solutions with "small" initial data near a stationary point, extending a well known result on the stable manifold in the case of a saddle point.

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Adaptivity and splitting for the numerical solution of SDEs with square-root diffusion.

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Presentation type: Talk

The Cox-Ingersoll-Ross (CIR) process is described by an Itô-type stochastic differential equation with a square-root diffusion and appears frequently in applications such as finance and neuroscience:

$$dX(t) = \kappa\left(\theta - X(t)\right)dt + \sigma\sqrt{X(t)}dW(t), \qquad X(0) = X_0 > 0, \tag{1}$$

Solutions of (1) are almost surely (a.s.) non-negative; in fact when $2\kappa\theta > \sigma^2$, a parameter constraint called Feller's condition, they are known to be a.s. positive.

Numerical methods for (1) produce a stochastic difference equation. The challenge is to control error induced by the discretisation in spite of the unbounded gradient of the diffusion near zero, and to preserve the a.s. positivity of trajectories. (1) is a useful test equation in this regard since techniques developed to handle the functional response near zero can be applied to more general equations.

We propose a domain invariant numerical method for (1) applied over both deterministic and adaptive random meshes and based upon a suitable transform followed by a splitting. Moment bounds and theoretical strong L_2 and L_1 convergence rates of order 1/4 the scheme are available in a restricted parameter regime. We then extend the new method to cover all parameter values by introducing a *soft zero* region (where the deterministic flow determines the approximation) resulting in a hybrid method that deals with the reflecting boundary.

From numerical simulations we observe an optimal convergence rate of 1 within the Feller regime. As σ increases and we move outside of this parameter region, we observe that the rates of strong convergence are competitive with other schemes in terms of convergence order, however the proposed method with adaptive timestepping consistently displays smaller error constants.

Allee plasticity rule in unsupervised learning environment

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Presentation type: Talk

The Allee effect was introduced by Allee (1949) and characterizes a phenomenon in population dynamics where there is a positive correlation between a population density or size and its per capita growth rate.The strong Allee effect occurs when a population has a "critical density" below which it declines to extinction while the weak Allee effect occurs when a population lacks such a "critical density", but at lower densities, the population growth rate arises with increasing densities. Brain plasticity can be thought of as the ability of the brain to adapt to external activities by modifying some of its synaptic structure. Synapses play an important role in the brain because they constitute junctions between nerve cells and therefore facilitate diffusion of chemical substances called neurotransmitters from the brain to other parts of the body. To understand these modifications at the functional and behavioral levels, one must understand how experience and training modify synapses and how these modifications change patterns of neuronal firings to affect behavior. In this talk, I will propose a discrete time model of brain plasticity based on the strong Allee effect that captures synaptic modifications at the functional level. Stability analysis of the model will be discussed and simulations will be given.

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An Accelerated Convex Optimization Algorithm with Line Searchand Applications in Machine Learning

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Talk:

In this talk, we introduce a new line search technique, then employ it to construct a novel accelerated forward–backward algorithm for solving convex minimization problems of the form of the summation of two convex functions in which one of these functions is smooth in a real Hilbert space. We establish a weak convergence to a solution of the proposed algorithm without the Lipschitz assumption on the gradient of the objective function. Furthermore, we analyze its performance by applying the proposed algorithm to solving classification problems on various data sets and compare with other line search algorithms. Based on the experiments, the proposed algorithm performs better than other line search algorithms.

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An Algebraic Approach to Solve a non Homogeneous Dynamic Cauchy-Euler Equation of High Order

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Presentation type: Talk

In this paper, we introduce a nonstandard method to find a particular solution for non-homogeneous dynamic Cauchy-Euler equations of high order. The proposed method uses new algebraic tools on discrete sets. This method provides an explicit particular solution for non-homogeneous dynamic Cauchy-Euler equations whose characteristic equations have distinct roots.

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An efficient numerical algorithm for pantograph delay differential equations

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Presentation type: 'Talk'

The aim of the study is to obtain solutions of some nonlinear pantograph delay differential equations using proposed algorithm, which is a combination differential transform and Faà di Bruno's formula. The proposed technique deals effectively with nonlinear terms present in the problem. The convergence results are discussed also. Three problems are investigated to show usefulness and efficiency of the proposed technique. Furthermore, the presented technique has good potential for applications to constant and time dependent delay differential equations.

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An unbounded system of rational difference equations with non-constant coefficients

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Presentation type: Talk

We establish the existence of unbounded solutions of the system

$$\begin{cases} x_{n+1} = \dfrac{a'_n}{B'_n y_n + C'_n x_n} \\ \\ y_{n+1} = \dfrac{b_n x_n}{C_n y_n} \end{cases} \quad \text{for } n = 0, 1, 2, \ldots,$$

with positive periodic coefficients, and positive initial conditions. This is an example of when "periodicity destroys boundedness," as all solutions of this system are bounded when the coefficients are positive constants. This family of systems is part of a larger family of systems difference equations, namely,

$$\begin{cases} x_{n+1} = \dfrac{a'_n + b'_n y_n + c'_n x_n}{A'_n + B'_n y_n + C'_n x_n} \\ \\ y_{n+1} = \dfrac{a_n + b_n x_n + c_n y_n}{A_n + B_n x_n + C_n y_n} \end{cases} \quad \text{for } n = 0, 1, 2, \ldots,$$

with $x_0 > 0$, $y_0 > 0$, and where the coefficient sequences, $\{a'_n\}$, $\{b'_n\}$, $\{c'_n\}$, $\{A'_n\}$, $\{B'_n\}$, $\{C'_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{A_n\}$, $\{B_n\}$, $\{C_n\}$, are both bounded above and below by positive constants or otherwise identically equal to zero. Further, we assume

$$(a_n + b_n + c_n)(a'_n + b'_n + c'_n)(A_n + B_n + C_n)(A'_n + B'_n + C'_n) > 0 \text{ for all } n,$$

to ensure that the denominators of the two parts of the system remain positive.

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Analysis of time fractional delay 2D-Navier-Stokes equations driven by colored noise

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Presentation type: 'Talk'

In this talk, we will show the local and global existence and uniqueness of mild solution to the time fractional 2D-Navier-Stokes equations with bounded delay driven by colored noise when the external force is nonzero. We emphasize for the global existence and uniqueness of mild solution, the model with higher regularity on the trilinear term is investigated since the singular kernel with our defined phase space cannot provide us appropriate estimates. We will also comment on the spatial discretization of the model.

References

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Analytic normalization of strongly hyperbolic (complex) Dulac germs

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Presentation type: Talk

Dulac germs are analytic germs on an interval $(0, d)$, $d > 0$, that can be analytically extended to certain complex domans called *standard quadratic domains*, admitting certain type of asymptotic expansion called *Dulac series*. More precisely, Dulac series are formal series of powers multiplied by real polynomials in the formal variable $\log z$, and they form a subset of the larger set of all *logarithmic transseries* (see [2]). Dulac germs appear as the first return maps of hyperbolic polycycles of analytic planar vector fields and they play an important role in the *Dulac problem* of nonaccumulation of limit cycles on a hyperbolic or semi-hyperbolic polycycle of an analytic planar vector field (see [3], [4]).

In this talk we consider *strongly hyperbolic Dulac germs* $f = z^\alpha + o(z^\alpha)$, $\alpha > 0$, $\alpha \neq 1$. We first define the formal composition of logarithmic transseries and prove that every *strongly hyperbolic logarithmic transseries* $\widehat{f} = z^\alpha +$"higher order terms", $\alpha > 0$, $\alpha \neq 1$, can be formally conjugated (normalized) to its leading term z^α via *parabolic conjugation* $\widehat{\varphi} = z +$"higher order terms". On the other hand, motivated by the classical *Böttcher Theorem* (see [1]), we prove that every analytic complex germ $f = z^\alpha + o(z^\alpha)$, $\alpha > 0$, $\alpha \neq 1$, defined on certain invariant complex domain, having certain logarithmic asymptotic bound, can be analytically conjugated to the analytic germ $z \mapsto z^\alpha$. In the end, we apply both results (the formal and analytic one) to prove that every strongly hyperbolic (complex) Dulac germ can be analytically conjugated (normalized) to the germ $z \mapsto z^\alpha$ via parabolic analytic conjugation $\varphi = z + o(z)$. Furthermore, the unique conjugation (normalization) φ is again a (complex) Dulac germ.

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Angular Values of Linear Discrete Time Dynamical Systems I

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Presentation type: Talk

In this presentation we introduce the new notion of angular values for a nonautonomous or random linear dynamical system. The angular value of dimension s measures the maximal average rotation which an s-dimensional subspace of the phase space experiences through the dynamics of a discrete-time linear system. This notion works for subspaces of arbitrary dimension s and avoids to specify orientation. Our main results relate the notion of angular values to the well-known dichotomy (or Sacker-Sell) spectrum and its associated spectral bundles. In particular, we prove a reduction theorem which shows that instead of maximizing over the whole Grassmannian, it suffices to maximize over so-called trace spaces which have their basis in the spectral fibers.

Part II of this talk shows how this theorem leads to a general algorithm for computing angular values of arbitrary dimension and how it can be utilized to derive explicit formulas for angular values in the autonomous case.

References

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Angular Values of Linear Discrete Time Dynamical Systems II

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Presentation type: Talk

In this talk we discuss applications of the general theory of angular values, explore its relation to rotation numbers, and analyze specific examples. For autonomous systems the general reduction theorem allows to reduce the computation to the two-dimensional case. If the latter has complex conjugate eigenvalues we employ ergodic theory to derive an explicit formula for the first angular value. Further, the reduction procedure serves as the basis for an algorithm to compute angular values of arbitrary dimension in nonautonomous systems. We apply the algorithm to systems of increasing complexity, notably to linearizations about a chaotic trajectory of a nonlinear system. A particular feature of the algorithm is to efficiently detect the fastest rotating subspace even if it is not dominant under forward dynamics. Finally, we indicate generalizations of our approach to random and continuous time dynamical systems.

References

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Appearance of closed invariant curves in a piecewise Cournot model with advertising

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Presentation type: 'Talk'

In the present paper we investigate two cases which can explain what happens when the Cournot equilibrium of a duopoly model loses stability through a Neimark-Sacker bifurcation of subcritical type. This kind of bifurcation involves complex dynamics which lead to the appearance of closed invariant curves. We analyze a Cournot model where competitors hold different plants and compete on advertising quantities. The model is described by a two-dimensional piecewise map in discrete time. Making use of analytical results and numerical simulations, we show that the appearance/disapearance of closed invariant curves is directly related to two different mechanisms, namely homoclinic bifurcations and border collision bifurcations.

References

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Asymptotic Characterisation of the Mean Square of Solutions of Perturbed Linear Stochastic Difference Equations

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Presentation type: Talk

In recent work, the authors have found necessary and sufficient conditions for the asymptotic stability in mean square of solutions of the finite order scalar linear stochastic difference equation

$$Y(n) = \left(\sum_{j=1}^{p} \alpha_j Y(n-j)\right) + \left(\sum_{j=1}^{q} \beta_j Y(n-j)\right)\xi(n), \quad n \geq 1. \tag{1}$$

In (1), $Y(n) = \psi(n)$ for $n \leq 0$, where ψ has finite second moment, and is independent of the iid (independent and identically distributed) zero mean and finite variance sequence ξ. This stability for (1) is relatively easy to check: it ensues if and only if the underlying deterministic equation (i.e. (1) with $\beta_j = 0$ for $j = 1, \ldots, q$) is asymptotically stable, and a certain contraction constant, which can be found by solving an auxiliary system of linear equations, is less than unity.

In this talk, we study the natural question: if (1) exhibits the "stability" mentioned above, when does this extend to solutions of the perturbed equation

$$X(n) = \left(\sum_{j=1}^{p} \alpha_j X(n-j) + f(n)\right) + \left(\sum_{j=1}^{q} \beta_j X(n-j) + g(n)\right)\xi(n), \quad n \geq 1,$$

where f and g are deterministic sequences in a "nice" space N (e.g., bounded, periodic, tending to zero, summable, square summable etc)? In the talk, we show that if (1) is mean square asymptotically stable, and f and g are in N, then generally the deterministic sequence $\mathbb{E}[X^2(n)]$ is in a nice space, which is very often N also. Furthermore, we give some converse results: for instance, if $\mathbb{E}[X^2]$ tends to zero for all choices of initial sequence ψ, then the unperturbed equation must be mean square asymptotically stable, and f and g must tend to zero also.

Bifurcation equations for periodic orbits of implicit discrete dynamical systems

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Presentation type: 'Talk'

Bifurcation equations, non-degeneracy and transversality conditions are obtained for the fold, transcritical, pitchfork and flip bifurcations for periodic points of one-dimensional implicitly defined discrete dynamical systems.

The backward Euler method and the trapezoid method for numeric solutions of ordinary differential equations fall in the category of implicit dynamical systems. Examples of bifurcations are given for some implicit dynamical systems including bifurcations for the backward Euler method when the stepsize is changed

References

[1] H. M. Oliveira, *Bifurcation equations for periodic orbits of implicit discrete dynamical systems*, Nonlinear Dynamics **91** (2018), 387 - 402.

Big or small? A new economic geography model with an endogenous switch in the market structure

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Presentation type: Talk

We present a new economic geography (NEG) model with a linear demand function where firms may change the perception of their relative dimension with respect to the local market. If they perceive themselves as big, they behave as Cournot oligopolists, otherwise they behave as monopolistic competitors. We first compare the pure cases in which only one market form prevails in the two-region economy. Comparing these pure cases of monopolistic and oligopolistic competition only a quantitative difference emerges. Subsequently, we assume that firms switch behaviour and start to interact strategically when the number of local firms is below a threshold; in that case, the market form evolves endogenously. Results change substantially. 'Break' and 'sustain' points are separated as in standard NEG models with an isoelastic demand function leading to a co-existence of equilibria. Stable partial agglomeration and oscillations with small amplitude are possible. The dynamics are described by a 1D piecewise smooth map, which can be continuous (in the pure monopolistic or oligopolistic cases) or discontinuous (when the market structure evolves endogenously). We analyse the bifurcation structure of the parameter space of the map comparing these cases. We show that the continuous maps have rather standard dynamics, while the discontinuous map is characterised by border collision bifurcations of fixed points and cycles, which lead to rich and complex bifurcation structures.

Bioeconomic model with the presence of tide effects in Moroccan coasts

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Presentation type: 'Talk'

The main objective of this work is the study of the effects of high tides and low tides on fishing effort, catches as well as profits in a bioeconomic model of populations of Sardina pilchardus, Engraulis encrasicolus and Xiphias gladius in Moroc-can areas. To achieve this objective, we studied the stability of the equilibrium points of our biological model then we added in our model the effect of the tides in the fishing effort which maximizes the profits of the fishermen under the constraint of the conservation of the biodiversity of these marine species using the generalized Nash equilibrium in the resolution of the bioeconomic model. As results, we were able to give the best fishing times according to the tides of each month of the whole year which will allow us to achieve better yields. Hence the importance of introducing the effect of high and low tides in bioeconomic models.

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Bistability-affected Period Adding in a Power Inverter with Hysteresis Control

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Presentation type: 'Special Session Talk'
The title of the special session: Invertible and Noninvertible Maps: Theory and Applications.

The development of power electronic switching converter systems has provided a broad range of new effective approaches to the conversion of electric power in many applications of renewable energy generation, transmission, and consumption. Today, there is a rapidly rising interest in the use of various forms of so-called hysteresis control in power electronics systems. The interest in hysteresis control is because the implementation of such a method is simple since it requires only relay elements (comparators). In addition, with the right choice of hysteresis control strategy, high quality of energy conversion is achieved, reduction of weight and size parameters of power filters, low harmonic distortion of output voltage or current, reduction of switching losses in power switches, increased efficiency and reliability. However, the possible dynamics of such systems are not yet well understood.

In the present work, we investigate the dynamics of a 1D map which acts as a model of an H-bridge inverter with hysteresis control. Due to the applied control strategy, the map may have several discontinuities inside the absorbing interval. Typically (although not always), the map is piecewise increasing on this interval and, similar to other piecewise increasing 1D maps, demonstrates a period adding bifurcation structure. Supported by experimental studies, we show that in the considered map this structure is partially affected by bistability, so that for some parameter values the absorbing interval of the map contains a single attracting cycle while for other parameter values a pair of attracting cycles of the same period exists. We show that the reason for possible bistability is related to the symmetry of the underlying model. Then, using symbolic dynamics, we explain which of the cycles are affected by bistability and which are not. This is done in terms of the associated symbolic sequence adding scheme (similar to the well-known Farey structure commonly used for rotation numbers) and its parent-child relationship. Proceeding in this way, we demonstrate that 2/3 of all attracting cycles in the considered system are bistability-affected.

Boundedness Character of a Family of First Order Rational Difference Equations With Nonconstant Coefficients

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Presentation type: Talk

We establish the boundedness character of first-order rational difference equations with bounded (but possibly not constant) coefficients. In particular, we establish the existence of unbounded solutions of systems of the form

$$\begin{cases} x_{n+1} = \dfrac{c'_n x_n}{B'_n y_n} \\ \\ y_{n+1} = \dfrac{b_n x_n + c_n y_n}{A_n + C_n y_n} \end{cases} \quad \text{for } n = 0, 1, 2, \ldots,$$

with coefficient sequences that are bounded above and below by positive constants except for $\{C_n\}$, which may be identically zero. This is an example of a family of systems in which all solutions are bounded when the coefficients are positive constants. This family is part of a larger family of systems of difference equations, namely,

$$\begin{cases} x_{n+1} = \dfrac{a'_n + b'_n y_n + c'_n x_n}{A'_n + B'_n y_n + C'_n x_n} \\ \\ y_{n+1} = \dfrac{a_n + b_n x_n + c_n y_n}{A_n + B_n x_n + C_n y_n} \end{cases} \quad \text{for } n = 0, 1, 2, \ldots,$$

with $x_0 > 0$, $y_0 > 0$, and where the coefficient sequences, $\{a'_n\}$, $\{b'_n\}$, $\{c'_n\}$, $\{A'_n\}$, $\{B'_n\}$, $\{C'_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{A_n\}$, $\{B_n\}$, $\{C_n\}$, are both bounded above and below by positive constants or otherwise identically equal to zero. Futher, we assume

$$(a_n + b_n + c_n)(a'_n + b'_n + c'_n)(A_n + B_n + C_n)(A'_n + B'_n + C'_n) > 0 \text{ for all } n,$$

to ensure that the numerators and denominators of the two parts of the system remain positive.

References

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Boundedness and global attractivity to three-species cyclic prey-predator of Volterra type difference equations

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Recently, S. Elaydi [2], S. Murakami [3] and Y. Raffoul [6] have studied the stability conditions of Volterra difference equations by using Liapunov methods. In this talk, we consider a sufficient condition for the globally asymptotic stability of a Volterra type difference equation is obtained by applying the technique of the Liapunov functional [2, 6]. On the other hand, Y. Oshime [5] has given a necessary and sufficient condition for every solution of the differential system to remain bounded as time passes. We now extend this result to three-species cyclic prey-predator of Volterra type difference equations, and moreover, we consider the existence of almost periodic solutions (cf. [1, 7]) to three-species cyclic prey-predator of Volterra type difference equations, by using this obtained boundedness theorem [4].

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Chaotic Neuron Model with Periodic Coefficients

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Presentation type: Talk (virtual)

We investigate the piecewise linear difference equation $x_{n+1} = \beta_n x_n - g(x_n)$, where in general case $\beta_n = \begin{cases} \beta_0, & \text{if } n = mk, \\ \beta_1, & \text{if } n = mk+1, \\ ... & \\ \beta_{m-1}, & \text{if } n = mk+(m-1), \end{cases}$ $k = 0, 1, 2, ..., \beta_n > 0$, $n = 0, 1, 2, ...$, where at least two of coefficients are different and $g(x)$ is in the form $g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases}$ This difference equation is prototype of one neuron model with internal decay rate β and signal function g. Authors investigated this model with periodic internal decay rate β_n with period two and three ([1, 2, 3]). Now we show that for certain values of coefficients β_n exists an attracting interval for which the model is chaotic. But if the initial value is chosen outside the mentioned interval, then the solution of the difference equation grows indefinitely (the limit is infinity).

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Complex Dynamical Behaviors in a Bertrand Game with Service Factor and Differentiated Products

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Presentation type: Talk

In this talk, taking the factor of product service provided by the manufacturers into consideration, a static Bertrand duopoly game with service factor is studied first, in which these two oligarchs produce differentiated products. A dynamic Bertrand duopoly game with bounded rationality is established by using the gradient mechanism. Keeping the adjustment speeds in a relatively small range will be conducive to the long-term stable operation of the two manufacturers. It is found that there is another 2-cycle, different from the one flip bifurcated from the fixed point, which may appear through a saddle-node bifurcation. The unstable set of the saddle cycle, connecting the saddle to the node, gives a closed invariant curve. In addition, the emergence of intermittent chaos implies that the established economic system has the capability of self-regulating, where PM-I intermittency and crisis-induced intermittency have been studied. With the help of the critical curves, the qualitative changes on the basin of attraction are investigated.

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Complex dynamics and bifurcations in a contest model with endogenous preferences

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Presentation type: Talk

Outcomes observed in laboratory experiments on contests are often not consistent with the results expected by theoretical models, with phenomena that frequently occur like overbidding or persisting oscillations in strategic choices. Several explanations have been suggested to understand such phenomena, dealing primarily with equilibrium analysis. We propose a dynamical model based on the coevolution of strategic choices and agent preferences. Each agent can have non self-interested preferences, which influence strategic choices and in turn evolve according to them. We show that multiple coexisting steady states characterized by non self-interested references can exist, and they lose stability as the prize increases, leading to endogenous oscillating dynamics. Finally, with an emphasis on two specific kinds of agents, we explain how overbidding can emerge. We show the occurrence of complex dynamics and we investigate the basins of attraction of multiple coexisting attractor.

Darboux and Bäcklund transformations for integrable difference equations

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Presentation type: Talk

In this talk we discuss Darboux and Bäcklund transformations for integrable difference equations and present a systematic method for their construction using Lax pairs. We derive their superposition principle, and explain the relation of the latter to Yang-Baxter maps. We demonstrate the implementation of our results using two illustrative examples, namely the Hirota KdV equation and an integrable discretisation of the NLS equation (aka Adler-Yamilov system).

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Degeneration and Homogenization of Fokker–Planck diffusion

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Presentation type: Talk

We investigate a diffusion equation of Fokker-Planck type [1]

$$\frac{\partial u_\varepsilon}{\partial t} - \Delta\left(b_\varepsilon u_\varepsilon\right) = f\,.$$

The material is inhomogeneous, since its diffusion coefficient b_ε oscillates rapidly with respect to the space variable x. We assume that the material contains a periodic array of holes of period ε. The size of the holes has instead order of $\eta\varepsilon$.

Moreover, we introduce the parameter δ which controls the magnitude of the coefficient b_ε inside the holes.

We investigate the onset of upscaled equations in the homogenization limit $\varepsilon \to 0$, when $\delta \to 0$ as well, namely when mass diffusion inside the holes becomes negligible.

In the framework of the standard Fick diffusion equation this problem is a rather classical one, we refer, for instance, to [2] and references therein.

Here we consider the Fokker–Planck diffusion equation, instead of the classical Fick one, performing a thorough study of all the possible cases obtained by tuning the relative rapidity at which δ, ε, $\eta\varepsilon$ tend to zero. Besides the mathematical interest of the issue, from a physical point of view we remark that we find different macroscopic equations when we assume different scalings for the parameters above.

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Detection of the first nonzero Lyapunov quantity in degenerate slow-fast Hopf bifurcations from fractality of planar contact points

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Presentation type: Talk

To determine complexity of a (degenerate) Hopf bifurcation one often has to put a system into normal form and find codimension (or the first nonzero Lyapunov quantity). This can be very challenging from computational point of view, and in this talk we present a new intrinsic fractal approach for a direct determination of the first nonzero Lyapunov quantity of a planar smooth slow-fast Hopf point or more general contact points. There is no need to use normal forms. Our appraoch is based on a fractal analysis of one-dimensional discrete dynamical systems generated by so-called entry-exit relation. This is an application of discrete dynamical systems to planar slow-fast systems. We also present simple fractal formulas for the determination of the first nonzero Lyapunov quantity. We focus on contact points of even contact order, odd singularity order and finite slow divergence. The study of this problem is initiated in [1] and completed in [2].

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Difference equations and optimal control in machine learning

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Presentation type: Talk

Any learned artificial neural network on a given set of observations represents a function of several variables with vector values or real values which satisfies some difference equation or discrete dynamic equation. However, in general it is unknown except very simply cases and we have trouble to tell anything about its properties behind very general results received from learning data. In applications, such as medicine, it needs to say not only that on training data we get some error, we have to know that an error is not greater than some ε for all data for which we consider the system. To this effect we develop an approximate optimal control approach for a set of neutral networks, parametrized by a set of controls and defined as difference equations. Moreover, to measure discrepancy, of the output of the network we define a functional into which we include probability distribution function estimating distribution of the data. We develop a dual dynamic programming ideas to formulate a new optimization problem. We apply it to derive and to prove sufficient approximate optimality conditions for approximate neural network which should work correctly for given ε with respect to built functional, on a data different than the set of observations.

Discrete impulses in difference-differential equations with generalized proportional Caputo fractional derivative and stability

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Presentation type: Talk

Impulses play an important role in many applications and models by difference equations ([5]) or differential equations([3, 7]). There are mainly to types of impulses in connection with their time of acting ([2]). In this talk we consider discrete type of impulses in the difference-differential equations with the generalized proportional Caputo fractional derivative is considered. The statement of the problem is discussed and set up. Several examples are presented to illustrate the connection between the discrete impulses and the continuous fractional differential equation. It is pointed out the deep influence of the discrete initially given times of the impulses on the fractional derivative. Some stability properties such as Ulam type stability are studied.

ACKNOWLEDGMENTS: This work is partially supported by the Bulgarian National Science Fund under Project KP-06-N32/7.

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Discrete probabilistic aggregative dynamics related to integer sequences

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Presentation type: Talk

We show how to obtain recurrences defining Catalan and Motzkin numbers recurrences by suitably defined discrete probabilistic aggregative dynamical systems.

Based on those examples we discuss general problem of interrelation between probabilistic dynamical systems and integer sequences.

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Discrete-Time Models and a SARS CoV-2 Mystery: Sub-Saharan Africa's Low SARS CoV-2 Disease Burden

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Presentation type: Talk

Worldwide, the ongoing SARS-CoV-2 virus has infected more than 444,000,000 people and killed more than 5,990,000. Conventional wisdom suggested that sub-Saharan Africa would bear a higher burden of COVID-19 infections and deaths for the same reason it suffers from other infectious diseases, such as malaria, cholera, HIV, Ebola, and so on. However, so far, sub-Saharan Africa has reported lower COVID-19 incidences and fatalities compared to the number of reported cases and deaths in developed countries. Existing continuous-time models of COVID-19 implicitly assume the availability of continuous stream of data for model parameterization and validation. However, for the ongoing COVID-19 pandemic, the disease data is typically reported at discrete-time intervals. As a result, data driven discrete-time model parameters can be related directly to the COVID-19 pandemic disease surveillance data without additional assumptions. In this talk, we argue that a discrete-time model would be a more appropriate modeling approach in the case of discretely reported data, while a continuous-time model would be more appropriate when the data are continuously reported.

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Dispersal-Driven Coexistence in a Multi-Patch Competition Model for Zebra and Quagga Mussels

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Presentation type: Talk

Zebra and quagga mussel are among the world's notorious invasive species because of their large and widespread ecological and economic effects. Although these two species have similar life histories and share many ecological traits, they have some significant ecological differences and impacts. Understanding their long-term population dynamics is critical to determining impacts and effective management. To investigate how the population reproduction rates, intraspecific and interspecific competitions, as well as dispersal abilities affect the population persistence and spatial distributions of the two species in a spatially heterogeneous environment, we develope a dynamic model that describes the competitive interactions between zebra and quagga mussels in multiple patches. The dynamic analysis of the model yields some sufficient conditions that lead to population and species persistence, extirpation, as well as to competitive exclusion and coexistence. By the numerical solutions of a two-patch model, we examine how the interplay between the local population dynamics in each patch and the individual dispersal between patches affects the competition outcomes of the two species in a spatially variable system.

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Dynamic Geometry Involving Kasner Polygons with Complex Parameter

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Keywords: dynamic geometry, Kasner polygons, complex coordinates.

For a real number α and an initial polygon $A_0^1 A_0^2 \dots A_0^k$, one can construct the polygon $A_1^1 A_1^2 \dots A_1^k$ such that A_1^1, A_1^2, $\dots$, A_1^k divide the segments $[A_0^1 A_0^2], [A_0^2 A_0^3]$, $\dots$, $[A_0^k A_0^1]$, respectively, in the ratio $1-\alpha : \alpha$. Continuing this process one obtains a sequence of polygons $A_n^1 A_n^2 \dots A_n^k$, $n \geq 0$, whose terms are called Kasner polygons (after E. Kasner (1878-1955)), or nested polygons when $\alpha \in (0,1)$. Many extensions involving non-constant weights have been studied (see, e.g., [3], [4], or [5]).

In this talk we present recent results concerning Kasner polygons, identifying the values of the complex parameter α for which the iterations are convergent, divergent, periodic, or dense, extending earlier results for triangles from [1] and [2].

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Dynamical Data to Difference Equations

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Presentation type: Talk

Experimenting on biological, physical, and artificial deterministic systems in order to generate a more informative dynamical model is a well-established practise in modern science. Although our ability to collect data has vastly improved, and data-driven and machine learning methods have exploded in popularity in recent years, the quest for the right observables, such as those offered by the famed Takens delay embedding theorem, has remained elusive.

Using a nonautonomous dynamical system, we demonstrate how to learn a function that assures precise reconstruction of the underlying dynamical system stably. We discover that the driven system renders observables that can *causally embed* a dynamical system [1, 2], a feature that will be detailed in the talk. The observables are universal similar to the delay coordinate maps being universal observables in Takens delay embedding. The role of the nonautonomous system is to bring about stability to noise or perturbations [3] that is absent in Takens delay embedding.

A result of the causal embedding is a topological conjugacy between the inverse limit system of the underlying system and the nonautonomous dynamics. Machine learning methods can then be used to learn accurate difference equations from data as a consequence of the topological conjugacy. Causal embedding-based models enable long-term consistency even for systems that have failed with previously reported data-driven approaches, in addition to stability and amenability for neural network based hardware implementations.

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Dynamics of Iteration Operator

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The semi-dynamical system of a continuous self-map is generated by iteration of the map, however the iteration itself, being an operator on the space of continuous self-maps, may generate interesting dynamical behaviors. In this talk, we characterize its fixed points and periodic points in the case that the compact metric space is a compact interval and prove that all orbits of the iteration operator are bounded but most fixed points are not stable. On the other hand, we prove that the iteration operator is not chaotic. This is a joint work with Murugan Veerapazham and Chaitanya Gopalakrishna.

Dynamics of Some Discontinuous Discrete Population Models

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Presentation type: Talk

Our aim is to compare and contrast the dynamics, in particular the permanence, periodicity, and oscillation of the certain classes of discontinuous discrete population models. We will focus on three models:

(i) The piecewise linear model introduced by Williamson [4]:

$$N_{n+1} = \begin{cases} \lambda^+ N_n, & \text{if } N_n \le K, \\ \lambda^- N_n, & \text{if } N_n > K, \end{cases}$$

where $0 < \lambda^- < 1 < \lambda^+$, $K > 0$, and $N_n > 0$ is the population size in generation n. This model appears to be equivalent to a piecewise linear equation, originated in the study of the West Nile epidemics:

$$x_{n+1} = (a - bh(x_n - c))x_n,$$

where $x_0 \ge 0$ and $a, b, c > 0$ such that $0 < b < 1 < a < b + 1$ and h is the Heaviside function [2].

(ii) The discontinuous Beverton-Holt type difference equation [3]

$$x_{n+1} = \frac{k(x_n) r(x_n) x_n}{k(x_n) + (r(x_n) - 1)x_n}, \quad n = 0, 1, ...,$$

where $x_0 > 0$, functions k and r are discontinuous piecewise constant functions $r(x) = R + Sh(x - T)$, $k(x) = K + Lh(x - M)$, satisfying $L, S \in \mathbb{R}$, $K > \max\{0, -L\}, R > \max\{1, -S + 1\}, T, M > 0$, and h is Heaviside function.

(iii) Finally we focus on the dynamics of nonlinear discontinuous population models exhibiting Allee type effects [1]. We will examine the model

$$x_{n+1} = F(x_n), \quad n = 0, 1, ...,$$

where $x_0 > 0$ and the function F satisfies the following hypotheses:

(H$_1$)

$$F(x) = \begin{cases} f(x), & \text{if } x \in [0, a) \\ g(x), & \text{if } x \in [a, b) \\ h(x), & \text{if } x \in [b, \infty) \end{cases} \tag{1}$$

such that $f \in C[[0, a], [0, \infty)]$, $g \in C^{\lceil\lceil\sim}, b], (0, \infty)]$, and $h \in C[[b, \infty), (0, \infty)]$.

(H$_2$) The functions f, g, and h are increasing on their respective domains.

(H$_3$) $f(x) < x$ for $x \in (0, a]$, $g(x) > x$ for $x \in [a, b]$, and $h(x) < x$ for $x \in [b, \infty)$.

$f(0) = 0$ and $\lim_{x\to\infty} h(x) = H > 0$.

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Dynamics of a delayed discrete size–structured chemostat with variable nutrient supply

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Presentation type: Talk

We study the dynamics inside a periodic and structured–population chemostat [3] containing a single nutrient and a microbial biomass whose behavior is described by the delayed and periodic system of difference equations,

$$\begin{cases} S_{t+1} &= E\,S_t^0 + S_t - f(S_t)(\mathbf{1}\cdot\mathbf{x}_t) - E\,[S_t - f(S_t)(\mathbf{1}\cdot\mathbf{x}_t)], \\ \mathbf{x}_{t+1} &= (1-E)A(S_{t-r})\mathbf{x}_t, \end{cases} \tag{1}$$

which is inspired in previous models developed by H. Smith [4], X.Q. Zhao [5] and J. Arino, J.L. Gouzé and A. Sciandra [1] for the autonomous and undelayed framework.

The chemostat is a spatially homogeneous bioreactor where a nutrient –denoted by S_t at time t– is consumed by a microbial species. In this context, the main contribution of [1, 4, 5] is to assume that the biomass of the microbial species –at time t– is divided in ℓ size classes denoted by $\mathbf{x}_t = (y_t^1, \ldots, y_t^\ell)^T \in \mathbb{R}^\ell$, which motivates the notation $\mathbf{1} = (1, \ldots, 1)$.

The nutrient is pumped into with rate ES_t^0, where $E \in (0,1)$ and the external nutrient S_t^0 is positive and ω–periodic. The consumption of nutrient is represented by a function $f(\cdot)$ whose properties will be detailed later, while the growth of the structured microbial biomass is modelled by the matrix $A(\cdot)$:

$$A(\cdot) = \begin{pmatrix} 1-P(\cdot) & 0 & \ldots & & & MP(\cdot) \\ MP(\cdot) & 1-P(\cdot) & 0 & \ldots & & 0 \\ 0 & MP(\cdot) & 1-P(\cdot) & 0 & \ldots & 0 \\ & & \ddots & & & 0 \\ 0 & \ldots & & & MP(\cdot) & 1-P(\cdot) \end{pmatrix},$$

where $M = 2^{\frac{1}{\ell}}$ and $P(\cdot) = \frac{f(\cdot)}{M-1}$, which shows the link between the processes of consumption and growth. As stated in [2], the matrix $A(\cdot)$ is an ubiquous tool in the study of structured population models and has distinguished properties: it is nonnegative for $S_t > 0$, circulant, semimagic, irreducible and primitive. These properties will play a key role to deduce the existence of ω–periodic solutions of (1).

The main differences of (1) with respect to [1, 4, 5] are: i) our assumption about the existence of a time interval $[-r, 0]$ necessary for nutrient intake to have an effect on the biomass growth, ii) we consider ω–periodic inputs S_t^0 instead of constant ones.

Our main result is the existence of ω–periodic solutions for (1). The main tools are a reduction of (1) to a planar system, in which the existence of ω–periodic solutions is obtained by means of the Poincaré translation operator and a topological fixed point theorem due to Browder. This planar result allow us to return to study (1) by using Floquet theory for linear difference equations and Perron–Frobenus Theorem.

In abscence of delays, we obtain a result of existence, uniquness and attractiveness of an ω–periodic solution. The main tools are results of exponential dicohotomy theory and Skew–product semiflows applied to Uniform persistence and stability.

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ECLSS: Extended Chaotic Map-based Certificateless Signature Scheme using Random Oracle Model

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Presentation Type: Talk

Chaos theory includes the study of the behavior of dynamical systems which are extremely delicate to complex constraints. Due to inherent traits of the chaotic system such as susceptibility to the initial condition, ergodicity, and systemic parameterization, it has been extensively used in cryptography. A certificateless signature scheme is a very functional technique that covered the flaws of an identity-based signature scheme. In this paper, a novel certificateless signature scheme (ECLSS) is proposed, by employing the chaotic properties. We validate that the proposed scheme is unforgeable against adaptive chosen message attack and proved the security by using the Random Oracle Model (ROM) under the computational hard Discrete Logarithm (C-DL) Problem that attains the required goal such as non-repudiation, confidentiality, and integrity. During the security analysis of the proposed scheme, we have shown that ECLSS satisfies many security attributes. Our ECLSS scheme is implemented with the widely accepted tool "AVISPA" and efficiency is demonstrated by fabricating comparisons in form of tables and graphical representation. In the performance analysis section, we show that the computational cost is reduced effectively by using extended chaotic property comparatively to the previous existing certificateless signature scheme.

Keywords: Certificateless signature, Chaos property, Discrete Logarithm problem, Diffie-Hallman assumption, Security and Privacy, Random oracle model.

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Endogenous preferences in a dynamic Cournot duopoly

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Presentation type: Talk

This research combines two strands of economic literature in a dynamic setting: endogenous preferences (on the consumer side) and strategic competitive markets (on the production side). This is done by considering a model in which aggregate demand depends on past consumption (Benhabib and Day, 1981; Gaertner and Jungeilges, 1988), whereas firms naively react to the rival's past decisions in determining aggregate supply (Puu, 1991). The interaction between the two sides of the market generates nonlinear and chaotic dynamics that could not be observed by considering each side separately. From an analytical point of view, the article shows that the stationary-state equilibrium can be destabilised either by flip bifurcations or by Neimark-Sacker bifurcations. Interestingly, the destabilisation of the stationary-state equilibrium can also occur when firms are homogeneous.

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Environmental sustainability, nonlinear dynamics and chaos reloaded: 0 matters!

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Presentation type: Talk

In this paper, reconsidering the overlapping generations (OLG) environmental model introduced in John and Pecchenino (1994) and Zhang (1999), and adopting a different specification for the environmental dynamics (as in Naimzada and Sodini, 2010), we analyze a dynamic model in which two different regimes may alternate: one in which the economy and the environment co-evolve in the same direction; the other in which the environmental problem is not internalized by the agents and where, therefore, they do not devolve any private resource to the environmental good, leading to a possible trade-off between environment and economic growth. The local and global analysis of the resulting piecewise defined map highlights that, either starting from a parametric configuration in which the dynamics evolve in the regime with positive contribution to the environmental good, or starting from a parametric configuration such as to determine dynamics with zero maintenance, the increase of the negative effect of the agents' consumption activity ends up generating dynamics in which the two regimes alternate, determining the arise of stable cycles or the occurrence of chaotic regimes. From a mathematical point of view, the occurrence of border collision bifurcations, able of generating sudden transitions from a stable cycle to a chaotic attractor, may be observed.

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ε-neighborhoods of orbits of time-one maps

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Presentation type: 'Talk'

In [2] we study the relationship between the multiplicity of a fixed point of a function g, and the dependence on ε of the length of ε-neighborhood of any discrete orbit of g, tending to the fixed point. We study the space of functions having a development in a Chebyshev scale and use multiplicity with respect to this space of functions. We introduce critical Minkowski order, the notion recovering the relationship between the multiplicity of fixed points and the dependence on ε of the length of ε-neighborhood of orbits. In [1] we introduce a parameter in the generator function g. Using the new notion of continuous ε-neighborhood of a discrete orbit we study saddle-node, transcritical and pitchfork bifurcation. We obtain the asymptotic expansion of the length of the continuous ε-neighborhood of a discrete orbit.

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Equilibrium and Stability Analysis of a Model of Immune Competition

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Presentation type: 'Poster'

A system of Ordinary Differential Equations "ODE" for population level statistics is studied. This model is derived from the partial integrodifferential system describing the dynamics of two active cells competing for a single resource. To be precise, we are interested in the evolution of the zeroth and first order moments, related to the activity of each population. This ODE model is analyzed for general resource uptake rate functionals and for certain important parameters, originating in the basic sub-models for individual birth, growth and death processes. The first part of this paper deals with the modelling including fundamental properties of solutions, while the second part develops the stability analysis for equilibrium solutions.

Key words: Population dynamics; Kinetic theory; Nonlinearity; Immune competition; Asymptotic analysis, Ordinary differential equations, Activity transfer, Coexistence.

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Discrete Fractional Boundary Value Problems

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Presentation type: Talk

Discrete fractional boundary value problems appear in numerous real world applications. It means that a deep understanding of these types of equations is necessary, and one of the most important concept is the existence of solutions to ensure the equation is useful. The focus of this talk is to present qualitative results regarding a general nonlinear discrete fractional boundary value problem with order between one and two. A few of the main results presented appear in a recent joint paper of [1] relating to an existence theorem, proving the existence of at least one solution to the boundary value problem, subject to validity of a certain key inequality that allows unrestricted growth in the problem.

The methodology used to accomplish these novel results use topological methods, plus two main techniques, firstly, a result showing that the solutions of the boundary value problem are exactly the solutions to a certain equivalent integral representation, and, second, the establishment of solutions satisfying certain a priori bounds provided the key inequality holds. Furthermore, several novel discrete fractional inequalities are presented, each of them interesting in itself and their usefulness are demonstrated in different contexts and boundary value problems. Finally, an example is presented alongside these current results, the new results and their versatility in other problems where the classical theory does not hold.

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Exterior, interior and expansion-like border collisions for chaotic attractors in 1D discontinuous maps

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Presentation type: Talk

In the present paper we consider a 1D piecewise linear map with *multiple* discontinuities and describe several bifurcations for chaotic attractors, which can not be associated with a homoclinic bifurcation of any cycle. Recall that border collision bifurcations of fixed points and cycles are widely investigated in contrast to those of chaotic attractors, for which their transformations are usually related to homoclinic bifurcations. For the most extensively studied class of piecewise smooth maps, i.e., 1D piecewise monotone maps with a *single* discontinuity, a chaotic attractor must include the border point, and thus, cannot collide with it.

Besides the "direct" border collision—when a chaotic attractor collides with a discontinuity point, which does not belong to this attractor—called an *exterior border collision bifurcation*, we also analyze more sophisticated cases. Some of them are grouped under the term *interior border collision bifurcations* that are related to disappearance of a preimage of a critical point (located inside a chaotic attractor). In addition, an *expansion border collision bifurcation* and *expansion-like* transitions of other type are reported.

Final Size of network SIR model and Katz-Bonacich centrality

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Presentation type: Talk

We study a deterministic system of ODE's for a network SIR model and we propose a discrete dynamical system that captures the long time behaviour of the network SIR model. With this discrete dynamical system we can approximate the long time behaviour of the solutions and find formulas that gives the node or edge that is most relevant to this long time behaviour. These formulas give a ranking of the nodes/edges that is special case of centrality measures studied in sociology/economy: The Katz-Bonacich centrality and intercentrality.

We apply these formulas to the problem of determining with real data which areas of a city should be intervened during an outbreak.

With data collected from mobile devices to approximate the contact network, and spatial information for the infected cases of the COVID-19 during the first semester of 2021 in the city of Bogota, we advised the local authorities of areas in Bogotá to be intervened with a pilot program. The pilot consisted of three areas to be intervened as wells as several areas to be used as control and consisted of different non pharmaceutical interventions used in the selected areas from July to October 2021.

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Fixed Point Results of Reich Contraction in Fuzzy Metric Spaces Endowed with Graph

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Presentation type:Talk

In this paper, we define a new class of Reich type contractions in the framework of complete fuzzy metric spaces satisfying the graph preserving conditions. A large number of different types of contractive mappings formulated using directed graphs in literature satisfy the presented contractive condition. Our main result is a natural generalization from fuzzy metric spaces to fuzzy metric spaces with a graph and enriches our knowledge of fixed points in such spaces. The results are further validated with the examples and application.

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Fractal analysis of degenerate spiral trajectories of a class of ordinary differential equations

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Presentation type: 'Talk'

We study the Minkowski dimension, also called the box dimension, of degenerate spiral trajectories of a class of ordinary differential equations. The singularities of focus type with two zero eigenvalues has been studied using the generalized polar coordinates and the discrete dynamical system generated by the Poincaré map. We obtain two cases, focus of type (n, n) having the same asymptotics of the Poincaré map in all directions, and the general (m, n) case. For the first case we compute box dimension, while for the second case we formulate a conjecture about the box dimension of a degenerate focus using numerical experiments. We also construct a three-dimensional vector field that contains a degenerate spiral, called an elliptical power spiral, as a trajectory. Degenerate (m, n) spirals typically arise in complex systems with an underlying dynamical process, such as elliptical whirlpools in a flowing body of water.

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Further properties in discrete time Lorenz model

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Presentation type: 'Talk'

In presentation, we investigate further properties of a two-dimensional noninvertible map $T : (x', y') = (x(1 + a\tau - y), (1 - \tau)y + \tau x^2)$ proposed by Lorenz in 1989, depending on the two parameters a and τ. We show the two different bifurcation scenarios occurring for $a > 0$ and $a < 0$. Two particular degenerate cases are investigated, at $\tau = 1$ and $\tau = 2$, describing the related bifurcations as a function of the parameter $a > 0$. For $\tau = 2$ a straight line filled with 2-cycles and the occurrence of a resonant case with rotation number $\pi/2$ in the Neimark-Sacker bifurcation of two fixed points determine particular bifurcations, associated with the existence of four invariant sets of the map. For $\tau = 1$ the critical curve degenerates into one point, the origin, which is also fixed for map T and focal point for the inverse map, leading to chaotic attracting sets with infinitely many lobes issuing from the origin. The transition to chaos from attracting closed curves is also analyzed, evidencing the occurrence of homoclinic tangles.

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Global dynamics and stability of discrete dynamical systems/difference equations and application to evolutionary population models

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In this talk, we provide an overview of the latest development on the challenging issues of the global dynamics and stability in discrete dynamical systems/difference equations. We start with monotone maps and then move to the more difficult non-monotone maps. One of the most mathematically challenging planar maps is the Ricker competition model where it is monotone only for certain specific values of the parameters. We investigate both cases when the Ricker model is monotone and when it is not monotone. Moreover, we investigate the case where the species are subject to evolutionary adaptations. Models of species under evolution is based on the postulates of Darwin. We investigate the local stability, and uniform persistence of species and show that persistence is enhanced by evolution. Finally, we develop the theory of mixed monotone maps and apply it effectively in the investigation of the global stability of evolutionary models.

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Global Dynamics in Discrete Versions of Differential Delay Models

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Presentation type: Talk

We study several aspects of global dynamics of certain difference equations with delay which can be interpreted as discretizations of continuous time differential delay models from applied sciences. We look into problems of local and global stability of equilibria, their instability and existence of periodic solutions, and complex/chaotic behaviors.

The difference equations under consideration include the following

$$\Delta x_n = -\mu x_n + f(x_n)\, g(x_{n-K}), \quad \Delta x_n = -F(x_n) + G(x_{n-K}), \quad n \in \mathbb{N},$$

where Δx_n is a difference operator (either forward, backward, or central), functions f, g, F, G are continuous real-valued ones, and $\mu > 0$ is a parameter. The first equation can be viewed as a discrete version of a continuous differential delay model of megakaryopoiesis [1], while the second equation can be treated as a discrete analog of corresponding differential-integral equations modeling some economic processes [2]. Both equations can also be tied to several other differential delay models from applications [3, 4]

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Graphon Dynamical Systems: a Law of Large Numbers, Large Deviations, and Applications

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Presentation type: Talk

We present a class of evolution equations, which are derived as a continuum limit of interacting dynamical systems on convergent graph sequences. For systems on random graphs, we present a dynamical law of large numbers and a large deviation principle. Furthermore, we discuss applications to the analysis of synchronization, pattern formation, and metastability in the Kuramoto model of coupled biological oscillators.

Grazing bifurcation and non-uniform hyperbolicity

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Presentation type: Talk

We study the dynamics of a vibro-impact system that is described by equations

$$\ddot{x} = f(t, x, \dot{x}, \mu) \qquad \text{if} \quad x_1 > 0 \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $n \geq 2$, $\mu \in (-\mu_0, \mu_0)$ is a one-dimensional real parameter. Suppose that the function $f : \mathbb{R}^{2n+1}$ is C^2 — smooth. Besides, we suppose that the following impact conditions are satisfied: $\dot{x}_1(t_0 + 0) = -r\dot{x}_1(t_0 - 0)$ provided $x(t_0) = 0$. This corresponds to a free-flight motion for $x_1 > 0$ and the threshold placed at $x_1 = 0$.

Of course, the thresholds of a more complicated shape can be considered, as well. Such an impulse-type dynamics cannot be described by differential equations only. This is an example of the so-called hybrid dynamics that combines some properties of discrete-time and continuous-time dynamics with some specific features one of which we are going to consider.

We study the so-called grazing bifurcation that corresponds to the following situation. There exists a continuous family of periodic solutions $\varphi(t, \mu)$ ($\mu \in (-\mu_0, \mu_0)$) of system (1) such that

1. $\varphi_1(0, \mu) = 0$ if $\mu > 0$;
2. $\varphi_1(t, \mu) > 0$ for $t \in (-\delta, \delta)$) if $\mu < 0$.

It was shown in [1] that a grazing bifurcation may imply chaos. Two different scenarios of chaotic dynamics in a neighborhood of grazing were described in papers [2] and [3]. In particular, it was shown that, for a near-impact solution, one Lyapunov exponent is big and positive while another one is big and negative.

Combining the ideas of these two papers, we developed a new techniques that allows us to write down asymptotic estimates for 'intermediate' Lyapunov exponents that allows us to check hyperbolicity for the periodic solutions of the obtained chaotic invariant set. Moreover, the dimensions of the corresponding stable and unstable manifolds can also be found.

And here, an interesting fact can be observed provided some additional conditions are satisfied. The initial periodic solution engenders a family of secondary ones. Some of these periodic solutions have a low-velocity impact while other ones do not. This means that dimensions of stable and unstable manifolds of the obtained periodic solutions differ and, therefore, the non-uniformly hyperbolic invariant set is there [4].

All in all, we discuss the conditions sufficient for a non-uniformly hyperbolic hyperbolic set appearance in a neighborhood of a grazing bifurcation. By now, this is seems to be one of the simplest known examples of dynamical systems that demonstrates such a sophisticated phenomenon.

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Hautus–Yamamoto criteria for approximate and exact controllability of difference delay equations

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Presentation type: 'Talk'

This talk presents results that can be found in the paper [1]. It deals with the controllability of finite-dimensional linear time-invariant difference delay equations, i.e., dynamical systems given at any time t by $x(t) = \sum_i^N A_i x(t-\Lambda_i) + Bu(t)$, where the state x and the control u belong to $\mathbb{R}^d$ and $\mathbb{R}^m$ respectively, the positive real numbers Λ_i's represent the (constant) delays and $A_1, \ldots, A_N, B$ are constant real matrices of appropriate size. Based on similar arguments given in the paper [2], we prove that the minimal time of controllability is upper bounded by $d \max_{1 \le i \le N} \Lambda_i$. The realization theory developed by Yamamoto for infinite dimensional systems [3] permits to obtain necessary and sufficient conditions expressed in the frequency domain for the controllability of these equations. The proof to obtain such results relies on the resolution of a Bézout identity over a field of Radon measures and the use of the Paley-Wiener theorem.

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How predators choose their prey to maximize their utility functions by using switching prey

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Presentation type: Poster

Abstract: In this work, we model the relationship between prey and predators by studying the interactive behavior of this prey-predator model and using the change of prey. The objective is to maximize the profit function of each predator by seeking the strategy provided by each predator to maximize its profit. To do so, we maximize this utility function being constrained by balance equations between biomass and trophic, and we show that this last problem is completely equivalent to finding the Generalized Nash Equilibrium Point. To calculate it, we use the conditions of KKT and we show that it is indeed a Problem of Linear Complementarity

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Hyperchaotic Attractors in the Model of Two Coupled Parabola Map

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Presentation type: Poster

We consider the system of two coupled one-dimensional parabola maps. It is well known that the parabola map is the simplest map that can exhibit chaotic dynamics, chaos in this map appears through an infinite cascade of period-doubling bifurcations [1]. For two coupled parabola maps we focus on studying attractors of two types: those which resemble the well-known discrete Lorenz-like attractors and those which are similar to the discrete Shilnikov attractors [2]-[4]. We describe and illustrate the scenarios of occurrence of chaotic attractors of both types.

We focus on some important aspects related to cascades of period-doubling bifurcations with periodic saddle points and bifurcations of the birth of an invariant curve. Also, we give more detailed analysis on the transition from Shilnikov-shape attractors to Lorenz-shape attractors through absorption of symmetric saddle periodic points, as well as mechanisms of hyperchaos emergence, including the appearance of multicomponent hyperchaotic Shilnikov-shape attractors.

This work is supported by the the Ministry of Science and Higher Education of the RF grant No. 075-15-2019-1931.

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Implicit linear difference equation over residue class rings

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Presentation type: Talk

Let $\mathbb{Z}_m = \mathbb{Z}/m\mathbb{Z}$ be a residue class ring modulo $m = p_1^{k_1} \dots p_r^{k_r}$, where $p_1, \dots, p_r$ are pairwise different primes and $k_1, \dots, k_r$ are natural numbers. Let $A, B, F_n \in \mathbb{Z}_m$ ($n \in \mathbb{Z}_+ = \{0, 1, 2, \dots\}$). Consider the linear difference equation

$$BX_{n+1} = AX_n + F_n, \quad n = 0, 1, 2, \dots, \tag{1}$$

over $\mathbb{Z}_m$. If B is non-invertible, this equation is said to be implicit.

The elements $a, b, f_n \in \{0, \dots, m-1\}$ are representatives of the corresponding residue classes $A, B, F_n \in \mathbb{Z}_m$. Denote the following greatest common divisor: $d = \gcd(a, b, m)$ and introduce $N = \prod\limits_{j\,:\,p_j \nmid b} p_j^{k_j}$ (if b is divisible by p_j for $j = 1, ..., r$, then $N = 1$).

Theorem 1. The following assertions hold.

1. The equation (1) has finitely many solution if and only if $d = 1$. Moreover, the amount of these solutions is equal to N.

2. The equation (1) has no solutions if and only if $d \nmid f_n$ for some $n \in \mathbb{Z}_+$.

3. The equation (1) has infinitely many solutions if and only if $d \neq 1$ and $d | f_n$ for all $n \in \mathbb{Z}_+$.

Corollary 1. The equation (1) has a unique solution if and only if B is nilpotent and A is an invertible element of the ring $\mathbb{Z}_m$.

We also proved the theorem about solvability of the initial problem $X_0 = Y_0 \in \mathbb{Z}_m$ for (1).

Input-Output Criteria for Stability of Discrete Dynamical Systems and Applications

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Presentation type: Talk

We present three categories of input-output methods for exploring exponential stability of discrete dynamical systems, starting from the techniques developed in [1, 2, 3]. We discuss the structures of the classes of sequence spaces from which one can select the input or output spaces and the minimal requirements in each case. Thus, we present complete characterizations of exponential stability of discrete dynamical systems from three perspectives, providing both global and local approaches. In addition, we give applications to the study of the robustness of exponential stability, based on some techniques initiated in [1, 5]. Finally, inspired by a method introduced in [4] we point out open problems and new directions.

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Insights on the Dynamics of a Piecewise-Linear Model for the Exchange Rate

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Presentation type: Talk

Starting from the stream of research on nonlinear economic models with heterogeneous agents, see, e.g., [3], [5], [4], [1] and references therein, we develop a stylized model of the foreign exchange market in which the exchange rate is driven by interactions between internationally operating firms, heterogeneous speculators and a central bank. In particular, we are interested in the effects of so-called targeting long-run fundamentals interventions according to which the central bank starts to intervene in the foreign exchange market when the exchange rate leaves a predefined target zone. Denoting by X_t the exchange rate at time t, the dynamics of the model can be represented by a two-dimensional piecewise linear map with three branches (see [2] for an overview of piecewise smooth dynamical systems):

$$\begin{cases} X_{t+1} = \begin{cases} (1+d-b_1+e)\,X_t - eY_{t-1} + b_0 & \text{if} \quad X_t \leq -Z \\ (1+d+e)\,X_t - eY_{t-1} & \text{if} \quad -Z < X_t < +Z \\ (1+d-b_1+e)\,X_t - eY_{t-1} - b_0 & \text{if} \quad X_t \geq Z \end{cases} \\ Y_{t+1} = X_t \end{cases} \tag{1}$$

where $d \in \mathbb{R}$ while for the other parameters we have $b_0 \geq -b_1 Z$, $b_1 \geq 0$, $e \geq 0$ and $Z > 0$. We carry out a detailed study of the main bifurcations and attractors of the model. The analytical results are useful to underline the main policy implications of the model. One remarkable finding is that a too aggressive central bank can set persistent exchange rate dynamics in motion by repeatedly triggering destabilizing speculative orders.

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Limit theorems for non-uniformly expanding maps under explicit conditions

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Presentation type: 'Talk'

The central limit theorem (CLT) and its variation has been extensively studied for random expanding dynamical systems in the past decades. In the past decade there was a significant progress for uniformly expanding dynamical systems, where results like local CLT and optimal CLT rates where obtained, as well as almost sure versions of the CLT . However, uniform expansion was crucial for the method of the latter result to work. Recently Dragičević and Sedro extended some of these results to expanding on the average random maps under a certain scaling condition on the underlying random potential which generates the random Birkhoff sums. In the talk I will show that for several classes of non-uniformly expanding maps (but not on the average), a certain type of explicit scaling condition is sufficient for the CLT and many other limit theorems.

Linear Dynamic-Algebraic Equations on Time Scales

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Presentation type: Talk

In this talk linear dynamic-algebraic equations on arbitrary time scales are introduced. They are defined admissible matrix sequences and the accompanying admissible projectors and characteristic values. It is deducted the Weierstrass-Kronecker form of dynamic-algebraic equations. The talk is provided with examples.

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Lyapunov Functions for Delay Reaction Diffusion Systems

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Presentation type: 'Talk'

Motivated by some biological and ecological problems given by a delay reaction diffusion system with Neumann boundary conditions; and knowing their associated Lyapunov functions for delay ordinary differential equations, we outline a method for determining their Lyapunov functions to prove the global stability. The method is based on adding integral terms to the original Lyapunov function for ordinary differential equations. The new approach is not general but it is applicable in a wide variety of delay reaction diffusion models with one discrete delay or more, distributed delay and combination of both of them.

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Lyapunov exponents for transfer operator cocycles of metastable maps: a quarantine approach

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Presentation type: Talk

We investigate the Lyapunov-Oseledets spectrum of transfer operator cocycles associated to a class of random metastable maps, indexed by a parameter ϵ, quantifying the strength of the leakage between two nearly invariant regions. We show that the system exhibits metastability, and identify the second Lyapunov exponent λ_2^ϵ within an error of order $\epsilon^2 \log \frac{1}{\epsilon}$. We show this approximation agrees with the naive prediction provided by a time-inhomogeneous two-state Markov chain. Furthermore, we show that $\lambda_1^\epsilon = 0$ and λ_2^ϵ are simple, and the only exceptional Lyapunov exponents of magnitude greater than $-\log 2 + O(\log\log \frac{1}{\epsilon} / \log \frac{1}{\epsilon})$.

Mathematical study of pollution negative effects of three scenarios : Applicated to Sardina pilchardus, Engraulidae and Phocoena phocoena marine populations.

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Presentation type: 'Poster'

During confinement, we noticed the appearance of different marine species, which is justified by the decrease in pollution rates, caused by fishing activities or recreational boats. This proves the importance of the mathematical analysis of biological models taking into account pollution effects. In the present work, we explored the effectiveness of such as study. Based on a discret biological model, we propose, analyse, modeling and simulating a predatory prey biological model. To make the study close to reality, we considered the populations Sardina pilchardus, Engraulidae and Phocoena phocoena. We search to show the impact of pollution rate variation on the evolution of these species. We thus implemented three scenarios aiming to illustrate the impact : one considering model with predator population and a null pollution mortality rate and two assuming the existence of preys populations and a non constant pollution mortality rates. As a main result, we founded that in the second scenario, even if we add two prey populations as favorable food for the predator, its biomass decreases from a certain rank, because of the increase in the pollution rate. Contrary to the rst scenario, which considerably elevated predator biomass despite the absence of preys.

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Method to Derive Discrete Population Models and their Continuous Counterparts

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Presentation type: Talk

We propose a derivation method to obtain discrete multi-species population models based on the assumption that the population sizes at time $t+1$ can be expressed as a multiple of the population sizes at time t. The multiplicative term is determined by placing the growth processes in the numerator and the decline processes in the denominator of the fitness of each population. Each resulting discrete model can be related to a continuous population model based on the same model assumptions using the relationship between discrete and continuous compounding in finance. We exploit this relationship to compare the stability for the continuous and the discrete models and argue that the corresponding continuous model analogues are *more stable*. We illustrate the derivation technique by providing several examples of discrete models of single and multi-species and compare their stability properties with their continuous analogues.

Model of evolution of the transmission of Chagas disease.

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Presentation type: 'Talk'

In this communication, we consider the infestation of a village by the domestic species *Triatoma. Dimidiata.* The village adjoins a forest representing the habitat of the vectors. The latter go to the village to feed. The food consists of a blood meal on the humans or mammals they raise. The transmission of *Trypanosoma Cruzi* from the vector to the host mainly takes place during this phase. In this work, we consider population of Tiatomines structured in time and space. Demography and spatial dispersal processes are captured by reaction-diffusion equations in two-dimensional space. In adequate functional spaces, the system of partial differential equations is transformed into an abstract differential equation. Our first objective is to show that the operator generates an analytic semigroup. We then prove the existence and uniqueness of a local solution to the corresponding Cauchy problem.

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Modified Mann-type iterative algorithm for solving fixed point and monotone problems with applications to image restoration problems

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Keywords: Mann-type algorithm, nonexpansive mapping, fixed point problem, image restoration problem, Hilbert space
Presentation type: 'Talk'

In this paper, modified Mann-type iterative algorithm combing with some inertial terms is created and studied for solving fixed point problem of a nonexpansive mapping in Hilbert spaces. Under some available properties on Hilbert spaces together with the considered mappings and the suitable assumptions on scalar controls, these allow us to prove strong convergence of a modified Mann-type iterative algorithm for solving a fixed point problem of a nonexpansive mapping which is related to a zero point problem of some monotone mappings. Finally, in order to show the value and usefulness of this newly discovered body of knowledge, we apply a modified Mann-type iterative algorithm to solve image restoration problems. Additionally, numerical results in various cases are shown in order to demonstrate the efficiency of our new algorithm, it was found that the new algorithm illustrates through numerical results that it has a higher performance than the previous existing results.

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Multistability in a stochastic consumption model

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Presentation type: ‘Talk’

We study the stochastic version of a model describing the dynamics of a two-consumer interaction [1] in discrete time

$$x_{1t+1} = \frac{b_1}{p_x p_y}(a_1 x_{1t}(b_1 - p_x x_{1t}) + D_{12} x_{2t}(b_2 - p_x x_{2t})) + \varepsilon \left(\xi_1 \sigma_1 + \xi_3 \sigma_3 \frac{b_1 D_{12}}{p_x p_y} x_{2t} \right) \tag{1}$$

$$x_{2t+1} = \frac{b_2}{p_x p_y}(a_2 x_{2t}(b_2 - p_x x_{2t}) + D_{21} x_{1t}(b_1 - p_x x_{1t})) + \varepsilon \xi_2 \sigma_2 \tag{2}$$

where x_1 and x_2 denote units of goods consumed by the first and second individual at time t, p_x and p_y are the unit prices of the commodities x and y, while the parameters b_1 and b_2 represent the income of the first and second individual. Prices and incomes are assumed to be constant in time. The real, strictly positive, parameters α_1, α_2 and D_{12}, D_{21} are referred to as *learning* and *influence* parameters respectively. The shocks ξ_1 and ξ_2 are assumed to be realizations of independently and identically distributed standard Gaussian random variates. The scalar constant $\varepsilon \geq 0$ functions as a noise intensity. The elements of the binary vector σ are used to model alternative types of noise. In particular, $(\sigma_1, \sigma_2, \sigma_3) = (1, 1, 0)$ is consistent with additive noise, while $(\sigma_1, \sigma_2, \sigma_3) = (0, 0, 1)$ refers to the case of parametric noise. We implement the indirect approach to studying stochastic dynamics.

In a first step, our research effort focuses on a parametric domain (D_{12}, D_{21}) for which the deterministic skeleton (non-invertible map) exhibits complex dynamics and multistability. Particular attention is paid to regions in which up to 5 attractors coexist. Considering D_{12} as our bifurcation parameter, we provide bifurcation scenarios and characterize basins of attraction by estimating uncertainty coefficients and fractal dimensions. Moreover, we discuss a scenario in which one of the attractors synchronizes the state variables x_1 and x_2.

On the second stage of the analysis, we scrutinize the effect of additive and parametric noise on the attractors of the system. In particular, noise-induced transitions between attractors are studied, and dominant attractors are identified. The phenomenon of de-synchronization caused by a random disturbance is described. For a constructive description of stochastic phenomena, the stochastic sensitivity function technique [2] and the associated confidence domain method are used.

The work was supported by the Ministry of Science and Higher Education of the Russian Federation (Ural Mathematical Center project No. 075-02-2022-877).

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Nonuniform Exponential Dichotomy and Admissibility

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Nonuniform exponential dichotomy describes nonuniform hyperbolicity for linear dynamical systems. In this talk, we will introduce our work on the relationship between admissibility of function classes without Lyapunov norms and nonuniform exponential dichotomy joint with Weinian Zhang and Davor Dragičević.

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Numerical modeling of dynamic systems describing contact problems with friction in elasticity

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The purpose of the article is to provide a correct (well-posed) formulation of dynamic contact problems with friction in elasticity, present the necessary and sufficient hypotheses for the existence and uniqueness of the solution, perform the discretization in space and time of equilibrium equations, and use regularization methods to avoid non-differentiable terms and convergent and numerically stable minimization algorithms for solving boundary optimal control problems. The novelty of the article is solving the problem of optimal control on the contact boundary, since the regularized problem of quasistatic contact with friction in elasticity fulfils the necessary conditions. This problem is of great importance in engineering applications (for example, for mobile robots and manipulator robots), where the control of forces and the control of displacements, as well as the control of sliding contact and/or fixed contact play a major role.

Numerical modeling of dynamic systems describing contact problems with friction in elasticity

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Presentation type:Talk

The purpose of the article is to provide a correct (well-posed) formulation of dynamic contact problems with friction in elasticity, present the necessary and sufficient hypotheses for the existence and uniqueness of the solution, perform the discretization in space and time of equilibrium equations, and use regularization methods to avoid non-differentiable terms and convergent and numerically stable minimization algorithms for solving boundary optimal control problems (see [1]-[5]). In order to be able to fulfill these desideratums it is useful to use discrete equations and discrete-time dynamics. The novelty of the article is solving the problem of optimal control on the contact boundary, since the regularized problem of quasistatic contact with friction in elasticity fulfils the necessary conditions. This problem is of great importance in engineering applications (for example, for mobile robots and manipulator robots), where the control of forces and the control of displacements, as well as the control of sliding contact and/or fixed contact play a major role.

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On 1:3 Resonance Under Reversible Perturbations of Conservative Cubic Hénon Maps

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Presentation type: Poster

We consider reversible non-conservative perturbations of the conservative cubic Hénon maps $H_3^{\pm}: \bar{x} = y, \bar{y} = -x + M_1 + M_2 y \pm y^3$ and study their influence on the 1:3 resonance, i.e. bifurcations of fixed points with eigenvalues $e^{\pm i2\pi/3}$. It follows from [1], this resonance is degenerate for $M_1 = 0, M_2 = -1$ when the corresponding fixed point is elliptic. We show that bifurcations of this point under reversible perturbations give rise to four 3-periodic orbits, two of them are symmetric and conservative (saddles in the case of map H_3^+ and elliptic orbits in the case of map H_3^-), the other two orbits are nonsymmetric and they compose symmetric couples of dissipative orbits (attracting and repelling orbits in the case of map H_3^+ and saddles with the Jacobians less than 1 and greater than 1 in the case of map H_3^-). We show that these local symmetry-breaking bifurcations can lead to mixed dynamics due to accompanying global reversible bifurcations of symmetric non-transversal homo- and heteroclinic cycles. We also generalize the results of [1] to the case of the $p:q$

resonances with odd q and show that all of them are also degenerate for the maps $H_3^{\pm}$ with $M_1 = 0$.

This work was supported by the grant of the Ministry of Science and Higher Education of the Russian Federation agreement No. 075-15-2019-1931 at the Laboratory of Dynamic Systems and Applications of the National Research University Higher School of Economics.

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On Qualitative Research of Ring Lattice of Neurons with Delayed Coupling on Time Scale

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Presentation type: Talk

This work is devoted to the modeling and investigation of the the delayed recurrent neural network in the form of ring lattice of n neurons on a time scale.

$$x_i^{\Delta}(t) = -x_i(t) + \delta g(x_{i+1}(\hat{\sigma}(t-\tau))), t \in \mathbb{T} \quad i(\mod n)$$

where $\delta \in \mathbb{R}$ is some constant, which can be presented in matrix form as follows

$$\begin{aligned} x^{\Delta}(t) &= -Ix(t) + Wg(x(\hat{\sigma}(t-\tau))), t \in \mathbb{T}, \\ x(t) &\in \mathbb{R}^n, \end{aligned} \tag{1}$$

where $I \in \mathbb{R}^{n \times n}$ is identity matrix and

$$W = \begin{bmatrix} 0 & \delta & 0 & 0 & \dots & 0 \\ 0 & 0 & \delta & 0 & \dots & 0 \\ 0 & 0 & 0 & \delta & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \delta \\ \delta & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

The problem of exponential stability is considered. For this purpose, Lyapunov-Krasovskii functional method of stability research on a time scale is used [1, 2, 3]. The results show us the effect of both time scale (graininess) and model (weights) parameters on qualitative behavior, which is changing from the stable focus to quasiperiodic limit cycles.

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On Target-Oriented Control of Hénon and Lozi maps

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Presentation type: 'Talk'

We explore stabilization for nonlinear systems of difference equations with modified Target-Oriented Control and a chosen equilibrium as a target, both in deterministic and stochastic settings. The influence of stochastic components in the control parameters is explored. The results are tested on the Hénon and the Lozi maps.

On bifurcations of Lorenz and Rovella attractors in the Lyubimov-Zaks model

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Presentation type: Talk

We provide numerical evidence for the existence of the Lorenz and Rovella (contracting Lorenz) attractors in the generalization of the Lorenz model proposed by Lyubimov and Zaks [1]. The Lorenz attractor is robustly chaotic (pseudohyperbolic) in contrast to the Rovella attractor [2] which is only measure-persistent (it exists for a set of parameter values, which is nowhere dense but has a positive Lebesgue measure). It is well known that in this model, for certain values of parameters, there exists a homoclinic butterfly (a pair of homoclinic loops) to the symmetric saddle equilibrium, which is neutral, i.e., its eigenvalues $\lambda_2 < \lambda_1 < 0 < \gamma$ are such that the saddle index $\nu = -\lambda_1/\gamma$ is equal to 1. The birth of the Lorenz attractor at this codimension-two bifurcation is established by means of numerical verification of the Shilnikov criterion. For the birth of the Rovella attractor, we propose a new criterion which is also verified numerically.

This work is supported by the the Ministry of Science and Higher Education of the RF grant No. 075-15-2019-1931.

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On discrete homoclinic attractors of three-dimensional diffeomorphisms

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Presentation type: Talk

In the theory of dynamical chaos, one of the most important and relevant is its direction that is associated with the study of strange attractors of multidimensional systems (with dimension of phase space ≥ 4 for flows and ≥ 3 for diffeomorphisms). Compared to the lower dimension, there are not too many meaningful results here, but almost all of them are of great importance for the theory of dynamical chaos. In our opinion, one of the most interesting results recently obtained in this direction are connected with the discovery of the so-called discrete homoclinic attractors [1–3]. By this term we primarily denote strange attractors of multidimensional maps (diffeomorphisms) that contain only one fixed point of saddle type and, hence, they also contain entirely its unstable manifold.

In the present talk we give a review on discrete homoclinic attractors of three-dimensional diffeomorphisms both orientable and nonorientable. We discuss the most important peculiarities of these attractors such as their geometric and homoclinic structures, phenomenological scenarios of their appearance, pseudohyperbolic properties etc.

We consider homoclinic attractors of various type such as discrete Lorenz attractors, discrete figure-eight attractors, discrete Shilnikov attractors etc. As illustrative examples, we will consider three-dimensional maps of the form $\bar{x} = y, \bar{y} = z, \bar{z} = Bx + G(y, z)$, that are called three-dimensional generalized Hénon maps.

This work was supported by the grant of the Ministry of Science and Higher Education of the Russian Federation agreement No. 075-15-2019-1931 at the Laboratory of Dynamic Systems and Applications of the National Research University Higher School of Economics. A. Gonchenko thanks Russian Science Foundation, grant 20-71-10048 (Numerical study of Shilnikov spiral attractors)

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On extension of fractional iterates of a Brouwer homeomorphism from a parallelizable region

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Presentation type: Talk

We discuss conditions enabling the extension of fractional iterates of a Brouwer homeomorphism defined on a maximal parallelizable region to the entire plane. We use properties of coverings of the plane with the maximal parallelizable regions of Brouwer homeomorphism and regularity conditions on the boundaries of these regions. The characterization of the set of irregular points of Brouwer homeomorphisms is also helpful.

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On impact of disturbance in the deployment problem of multi-agent system

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Presentation type: Talk

The class of Multi-Agent System (MAS) covers a generic family of dynamics composed of multiple interacting subsystems called agents. One of the main tasks while dealing with MAS is to design the control strategies for a group of agents in view of covering a known, predetermined target area in order to obtain a static configuration so that the region of coverage is maximized. This problem is known as the *deployment problem* or *coverage problem*. There are different approaches to the problem of deploying agents according the local or global information exchanged and the knowledge of the environment.

The present work is considering the decentralized deployment using a dynamic Voronoi partition. This method is built on the agents' current position and induces a control policy, which is nonlinear due to the agents arrangement. This means that at each time instant, a bounded convex polyhedron, which is the working environment, is partitioned using a Voronoi algorithm. In these schemes, the polytopic target environment is partitioned into a finite collection of polytopic Voronoi cells as there are agents. Moreover, it is necessary to designate internal target points where agents can reach a static configuration. For this purpose, we consider the Chebyshev centers which can be expressed geometrically the corresponding Voronoi cell.

Within this framework recent results show (see [1]) that nominal closed-loop dynamics are stable and converge to a consensus-like equilibrium. Our aim is to go beyond the state of the art and analyse the impact of additive disturbances on the multi-agent behaviour and the overall coverage problem. Such a robustness analysis is particularly useful in practice given the uncertainties available on the sensing and actuation channels that can be modelled in terms of additive uncertainties.

This work considers a one-dimensional case $\mathbb{R}$, i.e. the agents' work environment trivializes to the interval, on the discrete-time framework. The main purpose of the paper is to analyze the behaviour of agents for the system in the presence of disturbances. Since disturbance gains have a significant impact on the behaviour of the agents such as switching of their positions, the main focus is to establish a robust invariant set around the equilibrium of the nominal dynamics for a bounded additive uncertainty. Our consideration was conducted in two ways: as a theoretical analysis and a numerical one based on numerous simulations.

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On integrable partial difference equations in rational approximation/interpolation theory

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Presentation type: Talk

The Frobenius identities of the Padé approximation theory [2] can be identified with the discrete-time Toda lattice equation [5] of the theory of discrete integrable systems [4], together with its linear problem. Recently such connection with integrability was generalized [3] to the Hermite–Padé approximation. The relevant integrable system is the celebrated Hirota equation [6] supplemented by the Paszkowski constraint [7]. Almost the same equations appear also in the theory of multiple orthogonal polynomials [1]. I would like to present the above-mentioned connection and its generalization to the rational interpolation problems.

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On non autonomous Venttsel problems in fractal domains

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Presentation type: 'Talk'

Some results on non autonomous semilinear Venttsel problems, in irregular domains of fractal type are presented. We will investigate well posedness and regularity results of the weak solution. A fixed point argument, in suitable functional spaces, is used to prove existence and uniqueness results. These results rely on an ad hoc Nash inequality and on ultracontractive results of the associated family of evolution operators. Open problems will be discussed in view of the numerical approximation of this problem with a finite difference scheme in time and finite elemnet method in space.

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On sections of the generating series of the solution to the multidimensional difference equation

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Presentation type: Talk

Generating functions (or z-transformation) are a powerful tool in enumerative combinatorial analysis, and the study of their properties is of great interest, especially when it comes to a function belonging to one of the classes in the hierarchy proposed by Richard Stanley (see [7, 6]): rational $\subset$ algebraic $\subset$ D-finite.

A. De Moivre in [5] considered power (generating) series $F(z) = a_0 + a_1 z + ... + a_k z^k + ...$ with coefficients $a_0, a_1, ..., a_k, ...$, forming a recursive sequence, i.e. a sequence satisfying a relation $c_0 a_{m+p} + c_1 a_{m+p-1} + ... + c_m a_p = 0,\ p = 0, 1, 2...$, where $c_j, j = 0, \ldots, p$, are some constants. It turned out that such series always represent rational functions.

Let $\mathbb{Z}$ and $\mathbb{Z}_{\geqslant}$ be the set of integers and non-negative integers, and $\mathbb{Z}^n = \underbrace{\mathbb{Z} \times \ldots \times \mathbb{Z}}_{n \text{ times}}$ and $\mathbb{Z}^n_{\geqslant} = \underbrace{\mathbb{Z}_{\geqslant} \times \ldots \times \mathbb{Z}_{\geqslant}}_{n \text{ times}}$, and $\Delta = \{\alpha^0, \alpha^1, \ldots, \alpha^N\} \subset \mathbb{Z}^n_{\geqslant}$. We formulate the *Cauchy problem* (see [2]): find a function $f(x)$ that satisfies the equation

$$\sum_{j=0}^{N} c_j f(x - \alpha^j) = g(x), \quad x \geqslant m, \tag{1}$$

and coincides with some given initial data function $\varphi : \mathbb{Z}^n \to \mathbb{C}$ on $X_0 = \{x \in \mathbb{Z}^n_{\geqslant} : x \not\geqslant m\} = \mathbb{Z}^n_{\geqslant} \setminus \left(m + \mathbb{Z}^n_{\geqslant}\right)$:

$$f(x) = \varphi(x),\ \ x \not\geqslant m. \tag{2}$$

Multidimensional difference equations of this kind arise in the theory of digital recursive filters, as well as in combinatorial analysis, where they are called linear recursive relations (see [1]).

We consider *a special case*, when $\Delta = \{e^1, e^2, \ldots, e^N\} \subset \mathbb{Z}^N$ is a set of unit vectors in the standard basis, $e^j = (0, \ldots, 0, 1, 0, \ldots, 0)$, where 1 is only on the j-th position: $c_0 f(\lambda) + c_1 f(\lambda - e^1) + \ldots + c_N f(\lambda - e^N) = g(\lambda)$ for $\lambda \in \mathbb{Z}^N_{\geqslant} + e^1 + \ldots + e^N$ and the initial data function is $f(\lambda) = \varphi(\lambda)$ for $\lambda \not\geqslant m = e^1 + \ldots + e^N$.

Let cones K_p and L_p be spanned by vectors from $\Delta_1 = \{e^1, \dots, e^p\}$ and $\Delta_2 = \{e^{p+1}, \dots, e^N\}$ respectively. Since vectors in set Δ are linearly independent, each element λ of cone K can be represented as a unique sum of elements x and y from cones K_p and L_p: $K \ni \lambda = x + y, y \in K_p, x \in L_p$. Consequently, generating series $F(z)$ can be represented as a sum

$$F(z) = \sum_{x \in L_p} F(K_p; x; z)\, z^x, \text{ where } F(K_p; x; z) = \sum_{y \in K_p} f(x+y) z^y.$$

We call $F(K_p; x; z)$ as **a section** of generating series $F(z)$ (compare with [3]).

We derive recurrence relations for sections of generating series of solutions of multidimensional difference equations and prove an analog of the Moivre theorem for sections of such generating series.

With an appropriate choice of set Δ of steps, the sections of the generating series for the number of paths on the integer lattice will represent known sequences of polynomials (see [4]). For steps $\alpha^1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \alpha^2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ the number of the lattice paths satisfies the equation $f(x_1, x_2) - f(x_1 - 1, x_2 - 1) - f(x_1 - 2, x_2) = 0$, and the sections of the generating series yields the known sequence of Fibonacci polynomials: $F(0; z_2) = 1$, $F(1; z_2) = z_2$, $F(2; z_2) = z_2^2 + 1$, $F(3; z_2) = z_2^3 + 2z_2$, $F(4; z_2) = z_2^4 + 3z_2^2 + 1, \dots$.

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On some dichotomy concepts with growth rates for discrete time systems in Banach spaces

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Presentation type: Talk

The aim of this talk is to present some concepts of uniform dichotomy for linear discrete time systems in Banach spaces. We give some characterizations for the uniform dichotomy with growth rates and we obtain necessary and sufficient conditions for the particular cases of uniform exponential dichotomy and uniform polynomial dichotomy. Also, we emphasize the connections between the concepts mentioned above.

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On the RPI approximations of the mRPI set in the case of zonotopic disturbances

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Presentation type: 'Talk'

In this presentation we provide a robust positive invariance (RPI) outer-approximation of the minimal RPI (mRPI) set associated to linear dynamics in the case of disturbances bounded by a zonotope (i.e., the projection of a hypercube from a higher space).

We exploit the dual representation of polytopic sets (which zonotopes also enjoy) to provide both an inner (RPI after applying a scaling factor) and an outer approximation (RPI by construction) of the mRPI set.

We concentrate on discrete-time dynamics but present the continuous case as well. Furthermore, we analyze the complexity of representation (number of support hyperplanes or generator vertices) as a function of its precision (measured as the distance between inner and outer approximations).

Lastly, we compare this approach with others employed in the state of the art (which go the route of arbitrary precision via recursive Minkowski summations or analytic descriptions obtained at the price of limited accuracy).

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On the linearization of infinite-dimensional random dynamical systems

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Presentation type: Talk

We present a new version of the Grobman-Hartman's linearization theorem for random dynamics. Our result holds for infinite dimensional systems whose linear part is not necessarily invertible. In addition, by adding some restrictions on the non-linear perturbations, we don't require for the linear part to be nonuniformly hyperbolic in the sense of Pesin but rather (besides requiring the existence of stable and unstable directions) allow for the existence of a third (central) direction on which we don't prescribe any behaviour for the dynamics. Moreover, under some additional nonuniform growth condition, we prove that the conjugacies given by the linearization procedure are Hölder continuous when restricted to bounded subsets of the space. The talk will be based on the results obtained in [1].

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On the relation between growth conditions on some operator means and the behavior of the resolvent for bounded operators on Banach spaces

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Presentation type: 'Talk'

Let $\mathcal{X}$ be a Banach spaces, and $\mathcal{B}(\mathcal{X})$ be the Banach algebra of all bounded linear operators on $\mathcal{X}$. The present talk is concerned with sequences $\{T_n\}$ in the closed convex hull of the powers of a fixed operator $T \in \mathcal{B}(\mathcal{X})$ of the form

$$T_n = \sum_{j\geq 0} t_{nj} T^j, \quad t_{nj} \geq 0\,, \quad \sum_{j\geq 0} t_{nj} = 1, \tag{0.1}$$

and if $t_{nj} > 0$ for infinitely many values of j, the above sum converges in norm.

We shall denote from now on by $\kappa(T)$ the set of such sequences in $\kappa(T)$. This general type of operator mean has already been considered in the literature, but only under the additional assumption that T is power-bounded, that is $\{T^n\}$ is bounded in norm. The basic idea behind the results presented here is that a great deal of estimates, or ergodic theorems for specific operator means, continue to hold in a fairly general context than the power boundedness, and they can be deduced from either:

1) A regularity condition for $\{T_n\}$ or, 2) Conditions invariant to rotations.

The regularity condition considered here is that for some fixed, nonnegative integer n_0, we have $TT_n - T_{n+n_0} \to 0$ either strongly, or in the operator norm. It is used in order to exhibit useful isometries related to $\{T_n\}$ that are acting on quotient spaces of X.

By a rotational invariant condition we mean a growth restriction obeyed not only by the sequence $\{T_n\}$, but also by all "rotated" sequences $\{T_{n\lambda}\}$ obtained by applying the same convex combinations to the operator λT. More precisely, for T_n as in (0.1) we set

$$T_{n\lambda} = \sum_{j\geq 0} t_{nj}\lambda^j T^j, \quad |\lambda| = 1. \tag{0.2}$$

This type of condition brings complex analysis in the picture, a powerful tool in the study of bounded linear operators.

Our main theorem is a far-reaching generalization of a result of O. Nevanlinna [3]. More precisely, for operators $T \in \mathcal{B}(\mathcal{X})$ with peripheral spectrum $\sigma(T) \cap \mathbb{T}$ of arclength measure zero we derive an estimate for the powers T^n, under the assumption that for some sequence $\{T_n\}$ in $\kappa(T)$, both sequences $\{T_n\}$ and the backward iterate $\{T_n^{(-1)}\}$ satisfy uniform rotational invariant growth restrictions. We shall only mention some applications of this theorem. For example, when applied to (discretized) Abel means the result yields the following Nevanlinna-type theorem: If $u : (1, \infty) \to (0, \infty)$ is decreasing, then such a behavior of the resolvent $\|(T - \lambda I)^{-1}\| \leq \frac{u(|\lambda|)}{|\lambda|-1}, \quad |\lambda| > 1$, implies $\|T^n\| = o\left(nu\left(\frac{n}{n-1}\right)\right), \quad n \to \infty$.

The result applies to other means as well, in particular, to other functions of T. We pay special attention to the Cesàro means. We prove that certain rotational invariant growth conditions for Cesàro means of any order $p \geq 2$ are actually equivalent to a Kreiss-type condition. When $p = 2$, a special case of this result was proved by J. Strikwerda and B. Wade in [4]. Other similar considerations on the Kreiss resolvent condition were obtained by E. Berkson and T. A. Gillespie [2] for trigonometrically well-bounded operators. Also, pointwise convergence of such means, relating vectors in X and functionals in the dual space are obtained in [6].

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On the relationship between Lozi maps and max-type difference equations

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Presentation type: Talk.

This is a joint work with Prof. Antonio Linero-Bas based on [5]. Our aim is to revise a transformation that links generalized Lozi maps, namely,

$$x_{n+1} = \alpha|x_n| + \beta x_n + \gamma x_{n-1} + \delta, \tag{1}$$

where $\alpha, \beta, \gamma, \delta$ are real numbers with $\alpha \neq 0$, with max-type difference equations. In this sense, according to the technique of topological conjugation, we relate the dynamics of a concrete Lozi map with a complete uniparametric family of max-equations, and we apply this fact to investigate the dynamics of two particular families. Moreover, we present some numerical simulations related to the topic and, finally, we propose some open problems that look into the relationship established between generalized Lozi maps and max-equations.

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Other levels of chaos for Turing Machine's systems:

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Other levels of chaos for Turing Machine's systems: Please specify 'Talk'

Under certain notions of topological dynamics, it will be introduced certain levels of chaos to the study of Turing Machines from Kurka's approach. The idea of this presentation is to talk about some properties that may be important when replicating chaotic properties in such machines. Furthermore, the idea is to study machines that have undecidability caused by the presence of chaotic behavior, thus many different properties of the unpredictability of orbits will be included and will be related in many levels that may explain the property of undecidability. Finally, a pair of examples of these results will be shown in this presentation.

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Period incrementing and Milnor attractors for non autonomous families of flat top tent maps

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Presentation type: Talk

Period adding and period incrementing structures in the bifurcation scenarios of autonomous piecewise-smooth dynamical systems have been described in several works in the last few years, see [2] and references there in. It was observed in [1], that the introduction of a constant plateau in the map leads naturally to the appearance of Milnor attractors, This happens for parameters that may be described as accumulation points of parameters related with period incrementing sequences. In this work we consider families of nonautonomous dynamical systems generated by the sequential iteration defined by a binary sequence s, of two flat top tent maps. We describe period incrementing structures in the bifurcation scenarios and show how Milnor attractors emerge as accumulations of period incrementing. We study the dependence of these phenomena on the iteration pattern s

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Periodic and quasi-polynomial solutions of implicit linear difference equations over commutative rings

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Presentation type: ‘Talk’

Let R be a commutative ring with identity and $a, b, f_n \in R$ $(n = 0, 1, 2, ...)$. Consider the following linear difference equation over R:

$$bx_{n+1} = ax_n - f_n, \quad n = 0, 1, 2, ... \tag{1}$$

If b is non-invertible, this equation is said to be implicit. We present results about existence and uniqueness of periodic or quasi-polynomial solutions of (1) over R.

Theorem 1. Let R be an integral domain. Then for any m-periodic sequence $\{f_n\}_{n=0}^{\infty}$ Equation (1) has a unique m-periodic solution if and only if $a^m - b^m$ is invertible in R. This solution has the form $x_n = (a^m - b^m)^{-1} \sum\limits_{j=0}^{m-1} b^j a^{m-1-j} f_{n+j}$, $n = 0, 1, ..., m-1$.

Theorem 2. Let R be a local Noetherian domain with characteristics zero and $\mathcal{M}$ is the maximal ideal of R. Suppose $\lambda, a, b, p_j \in R$ $(j = 0, ..., s)$, $\lambda \neq 0$, a is invertible and b is a non-invertible element of R. Then there exists a unique solution over R of the equation $bx_{n+1} = ax_n - \lambda^n \sum\limits_{j=0}^{s} p_j n^j$, $\quad n = 0, 1, 2, ...$ It has the form

$$x_n = \lambda^n \sum_{k=0}^{\infty} a^{-k-1} b^k \lambda^k \sum_{j=0}^{s} p_j \sum_{r=0}^{j} C_j^r k^{j-r} n^r, \quad n = 0, 1, 2, ...,$$

where all the series converge in the ring R with respect to the $\mathcal{M}$-adic topology.

Periodic maps of mixed monotonicity

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Presentation type: 'Talk'

We consider periodic difference equations of the form $x_{n+1} = F(n, x_n, x_{n-1})$ in which F is p-periodic in n, non-deceasing in x_n and nonincreasing in x_{n-1}. We introduce the concept of artificial cycles and give illustrative examples. Then, we generalize some existing global stability results. In particular, we show the following result: Suppose the positive cone is invariant, there exists a point $\xi_0 = (A, B) = ((a, b), (c, d))$ such that $a < c, d < b$ and $\Omega := \{X : \quad X \in \mathcal{O}^+_{F_n}(X_0),\ A \leq_{se} X_0 \leq_{se} B\}$ is pre-compact. If $\xi_0 \leq_{se} G_{p-1} \circ \cdots G_1 \circ G_0(\xi_0)$, where $G_j(x, y, u, v) := (F(x, y), u, F(u, v), x)$, and F has a unique q-cycle but no artificial cycles, then q is a divisor of p and the q-cycle is globally attracting with respect to Ω.

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Quenched linear response for expanding on average cocycles

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Presentation type: Talk

After recalling what is linear response in a deterministic setting, I will explain how one can generalize this notion for a class of non-autonomous system, namely random compositions of uniformly hyperbolic maps [1, 2], and present recent results obtained with Davor Dragičević and Paolo Giulietti [3], where we establish a linear response result under a weaker expanding on average condition, which allow us to choose randomly among families containing neutral or contracting maps.

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Random-Like properties of Chaotic forcing

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Presentation type: Talk

We prove that skew systems with a sufficiently expanding base have approximate exponential decay of correlations, meaning that the exponential rate is observed modulo an error. The fiber maps are only assumed to be Lipschitz regular and to depend on the base in a way that guarantees diffusive behaviour on the vertical component. The assumptions do not imply an hyperbolic picture and one cannot rely on the spectral properties of the transfer operators involved. The approximate nature of the result is the inevitable price one pays for having so mild assumptions on the dynamics on the vertical component. However, the error in the approximation goes to zero when the expansion of the base tends to infinity. The result can be applied beyond the original setup when combined with acceleration or conjugation arguments, as our examples show. Joint work with S. Marmi (SNS) and M. Tanzi (NYU - Courant Institute).

Rate of convergence in the disjunctive chaos game

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Presentation type: Talk

It is known that an attractor of a contractive iterated function system is the omega-limit of the orbit that is driven by a disjunctive sequence (i.e., a sequence of symbols, which contains all possible finite words). In particular, this convergence holds with probability 1, when the orbit is driven by a sequence generated by a chain with complete connections with positively minorized transition probabilities, the most simple case being a Bernoulli scheme. Very recently, Bárány, Jurga and Kolossváry have established the rate of convergence of the probabilistic chaos game in terms of the box dimension, cf. [1]. We will present what happens to the rate of convergence when a disjunctive chaos game is considered instead of the probabilistic one, cf. [2].

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Rational-Linear Anticompetitive Systems of Difference Equations

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Presentation type: Talk

There are 112 systems of two rational-linear difference equations where, for all positive values of the parameters, the corresponding map is anticompetitive and the square of the map is strongly competitive. We present a theorem that describes the global behavior of the solutions for all 112 systems.

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Real exponential asymptotic behaviour is generic in the mean square of two-dimensional linear stochastic difference equations

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Presentation type: Talk

In this talk we are concerned with the mean square behaviour of linear 2-dimensional systems of stochastic difference equations with constant coefficients. The behaviour of such systems has been studied in great detail over the past fifty years and in particular a full characterisation of the mean square of n-dimensional systems was given by Richard Bellman [1]. Using Bellmans's approach, one can show the mean square process obeys an $n^2 \times n^2$ linear system of deterministic difference equations, hence all information regarding the mean square process can be inferred by studying the eigenstructure of the associated $n^2 \times n^2$ coefficient matrix. For high dimensional problems finding exact evolutionary behaviour becomes highly intractable but for the case where $n = 2$, such computational issues can be surmounted and it is such eigenvalue analysis to which this talk is devoted. In particular we show that for arbitrarily small and large noise the dominant dynamics are **always** real exponential where dominance is classified by the long term behaviour of solutions of the mean square process. We also identify special cases wherein real exponential behaviour prevails independent of the level of noise introduced into the system. Our approach is to make a suitable coordinate transformation which results in a reduction of complexity in the characteristic polynomial of the associated 4×4 matrix. It should be noted we make no prior assumptions on structure of the underlying coefficient matrices to allow for a completely general treatment of the mean square. It is of particular interest that upon introduction of arbitrarily small noise into the underlying deterministic system, solutions are **unable** to produce oscillatory behaviour even in the case when the solution of the noise-free equation has oscillatory solutions. The remarkable rarity of dominant oscillatory solutions, regardless of the eigenstructure of the drift and diffusion matrices in the two-dimensional case, leads to the conjecture that the phenomenon of dominant real exponential behaviour in the mean square may extend to arbitrarily many dimensions. However, the methods of proof used in this talk would not readily generalise to attack this conjecture. These are preliminary results in part of a larger study with Conall Kelly (UCC) on the seemingly generic appearance of dominant real exponential asymptotic behaviour in the mean square in autonomous linear stochastic equations.

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Recent Progresses on C^1 Linearization of Hyperbolic Dynamical Systems

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Presentation type: Talk

Smooth linearization is an important theory in the study of dynamical systems. In this talk, we will introduce some recent progresses on hyperbolic C^1 linearization, including the problems of sharp smoothness and weak spectral conditions, as well as the generalizations of classical results to infinite dimensional, random and nonautonomous cases. In particular, the difficulty of no bunching condition will be overcome when we prove the C^1 smoothness of invariant foliations. These are joint works with Weinian Zhang, Kening Lu, Witold Jarczyk and Davor Dragičević.

Recurrence Coefficients of Orthogonal Polynomials and Painlevé Equations

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Presentation type: 'Talk'

It is well known that Painlevé equations, both differential and discrete, often appear in the study of various orthogonal polynomials. For example, discrete Painlevé equations describe the dependence of the coefficients of the three-term recurrence relations as functions of the degree n, whereas differential Painlevé equations sometimes describe the dependence on some continuous weight parameters. I will present an explicit step-by-step procedure of reducing discrete or differential Painlevé equations that appear in applied problems to their standard form by finding some appropriate change of coordinates using the geometric approach and give examples. This is a joint work with A. Stokes (Japan) and A. Dzhamay (USA).

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Recursions and evolutions for the zeros of some entire functions

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Talk

We present some observations on the distribution of the zeros of entire functions, starting from the Airy's functions. A recursive sequence solved by the zeros is presented. The dependence of the position of the zeros is related to the initial conditions of the corresponding ODE. We show how, with the introduction of parameters, it is possible to get a system of infinitely many coupled nonlinear PDEs describing how the zeros evolve and exhibiting interesting properties. Some numerics will be given.

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Reduction of Systems of Linear 2-D Difference Equations to a Single Equation

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Presentation type: Talk

A 2-D system is a system in which information propagates in two independent directions. These can be space/time or space/space directions. Such systems may be represented by bivariate polynomial matrices over an arbitrary field. The problem of reducing a linear system of functional equations to one containing fewer equations and unknowns was first studied by the French mathematician J.P. Serre [7]. In this paper the concept of Smith form of a polynomial matrix is used to establish necessary and sufficient conditions under which an underdetemined linear system of 2-D difference equations is equivalent to a system containing only one equation in one unknown. The reduction involved must of course preserve relevant system properties from the systems theory point of view. The reduced equivalent representation simplifies in general the study of such systems.

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Riordan arrays and Difference equations for restricted lattice paths

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Presentation type: Talk

We use some combinatorial methods to study lattice paths on the positive octant. We introduce an identity which automatically gives some counting generating functions for the restricted lattice path problems and also we study some lattice paths associated with lower triangular arrays. An infinite lower triangular matrix $F = (f_{x,y})_{x,y\geqslant 0}$ associated to this kind of array is a Riordan array where the columns of the matrix are coefficients of certain formal power series and we obtain new combinatorial interpretations for some Riordan arrays in terms of restricted lattice paths.

Theorem. Let $F(x, y)$ be the set of all enumerated lattice paths starting at $(0, 0)$ and ending at the point $(x + y(m - 1), x + y(n - 1))$ that use the finite set of two general steps $\mathfrak{B} = \{(n, m), (m, n)\}$ where $n \in \mathbb{Z}_{\geqslant}, m \in \mathbb{Z}_{+}$, for $0 \leqslant n < m$ and stay on or below the main diagonal $y = x$. Then the infinite lower triangular array $(f_{x,y})_{x\geqslant y\geqslant 0}$ can be represented by a Bell-type Riordan array as $F = (F(t), tF(t))$, where $F(t) = \sum_{x=0}^{\infty} f_{x,0}t^x = \dfrac{1-\sqrt{1-4t^{m+n}}}{2t^{m+n}}$ is the generating function.

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Sentiment-driven financial market dynamics: Mathematical insights from a 2D nonsmooth map

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Presentation type: Talk

We develop a simple financial market model in which a market maker adjusts the price with respect to orders placed by chartists and fundamentalists. A novel feature of our model is that fundamentalists optimistically/neutrally/pessimistically believe in a relatively high/unchanged/low fundamental value when the financial market is relatively increasing/stable/decreasing. As it turns out, the dynamics of our model is driven by a two-dimensional discontinuous piecewise linear map

$$T : \begin{cases} P_{t+1} = & \begin{cases} (1+b-c)P_t - bX_t + d & \text{if } (P_t - X_t) > h \\ (1+b-c)P_t - bX_t & \text{if } -h \leq (P_t - X_t) \leq h \\ (1+b-c)P_t - bX_t - d & \text{if } (P_t - X_t) < -h \end{cases} \\ \\ X_{t+1} = & P_t \end{cases} \tag{1}$$

where b, c, d and h are parameters, P_t is the main variable, $X_t = P_{t-1}$.

For $h = 0$, sentimental traders are either optimistic (sufficiently rising prices) or pessimistic (falling prices). Map T is defined by three linear maps: T_L below the diagonal, T_U above the diagonal, and T_O on the diagonal. For this case, we provide an in-depth analytical and numerical investigation [1]. Among other things, we obtain in explicit form the boundaries of the periodicity regions associated with attracting cycles with rotation number $1/n$, $n \geq 3$. These boundaries correspond to border collision bifurcations of the related cycles. We show that the periodicity regions are organized in a specific period adding structure [2], and some of the regions may overlap. Several examples of coexisting cycles and their basins of attraction are also presented. Note that for $h = 0$, symbolic representation of a generic cycle consists of two letters, L and U.

Then we extend the obtained results to the case $h > 0$, when sentimental traders may also be neutral (if prices are relatively stable). In this case, map T_O is defined not just on the diagonal as it holds for $h = 0$, but on the strip $x-h < y < x+h$. This leads to a new type of generic cycles whose symbolic representation includes three letters, L, U and O. We show that corresponding bifurcation structure overlaps with the one related to the cycles with only two letters, L and U, in their symbolic sequences.

Economically, our results offer a new explanation for the boom-bust behaviour of actual financial markets.

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Snap-back repeller and Shilnikov singular chaotic attractor in the simplest neuron model

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Presentation type: Talk

In the present talk we study a dynamical behavior of neuron model in for map. The simplest map in which typical behavior of neural models can be observed must be two-dimensional, since should have two time scales. Two-dimensional maps can demonstrate a wide variety of dynamic modes, including the birth of an invariant curve, chaos, hyperchaos [1]. In the frame of this work, we turn to numerical simulation of the dynamics of the Chialvo map which was introduced for the first time in [2].

The Chialvo model was proposed as the simplest map in which one can observe burst-spike dynamics, including chaotic ones. This map is written as follows:

$$\begin{aligned} x_{n+1} &= x_n^2 exp(y_n - x_n) + I, \\ y_{n+1} &= ay_n - bx_n + c, \end{aligned} \tag{1}$$

Here the variable x reproduces the dynamics of the membrane potential; y is a restoring variable; a, b, c, I are parameters. These parameters control the dynamics of the system as follows: a is the recovery time constant ($a < 1$); b determines the degree of dependence of restorative processes on the level of activity; c is a constant offset; parameter I characterizes the action of ion currents injected into the neuron. Thus, model (1) is a two-dimensional map with four control parameters. This map is an endomorphism, i.e. is non-invertable.

We studied model (1) and revealed that there is possible formation of singular chaotic Shilnikov attractor. For a two-dimensional map, the Andronov-Hopf bifurcation is possible, as a result of which the fixed point loses stability and an invariant curve is born, while the fixed point turns into an unstable focus. For endomorphism, a singular mechanism of returning the phase trajectory to the vicinity of the unstable focus can be implemented, which, when it is absorbed by a chaotic attractor, leads to the formation of a singular chaotic Shilnikov attractor. Such mechanism of returning the phase trajectory known as snap-back repeller [3]. In our talk we will show different type of singular chaotic Shilnikov attractors and discuss they features.

The work is supported by the Russian Science Foundation (Project 20-71-10048).

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Solutions of Generalized Difference Equations and its Applications

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Presentation type: Talk

Abstract

In this talk, we present the generalized difference equation of higher kind and derive the discrete version of Leibnitz Theorem, Binomial Theorem and Newtons Formula according to its operator. Also, discuss the virus population problem using the solutions of generalized difference equations is an one of the applications in the field of Mathematical Biology. In addition, Verification of the controllability and observability for the linearized model and the estimation of correct state variables derived from the response of the discrete state variables to their control inputs in Permanent Magnet Stepper Motor using generalized difference equations in the field of Electrical Engineering.

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Some results about p-cycles of third order

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Presentation type: 'Talk'

In our talk we present some results appearing in [4], a preprint written jointly with Daniel Nieves Roldán, from the University of Murcia. In turn, this work supposes a natural continuation of [1].

We will deal with the notion of global periodicity, namely, we study classes of difference equations having the property that all their solutions are periodic. In particular, we analyze the possible global periodicity for difference equations of type $x_{n+3} = x_i f(x_j, x_k)$, with $i, j, k \in \{n, n+1, n+2\}$ pairwise distinct, where $f : (0, \infty)^2 \longrightarrow (0, \infty)$ is a continuous map and the initial conditions are positive. We prove that if f is symmetric ($f(x, y) = f(y, x)$ for all $x, y > 0$), then no 6-cycles are possible in this frame. However, if f separates variables, namely, $f(x, y) = g(x)h(y)$ with $g, h : (0, \infty) \longrightarrow (0, \infty)$ continuous maps, then the unique 6-cycle is given by $x_{n+3} = x_n \left(x_{n+2}/x_{n+1}\right)^2$. We will finish our talk by presenting some open questions related with the global periodicity of this class of third order difference equations.

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Stability analysis of two-term fractional-order difference equations

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Presentation type: Online Talk

Stability properties are investigated for fractional-order difference equations of Caputo-type with two subunitary fractional orders. Consider the following two-term fractional-order difference equations with Caputo forward difference operators:

$$^{c}\Delta^{q_1}x(nh) + a^{c}\Delta^{q_2}x(nh) + bx(nh) = 0, \tag{1}$$

where h is the discretization step and $0 < q_1 \leq q_2 \leq 1$ are the fractional orders of Caputo forward difference operators.

The stability and instability conditions for the considered equations are explored both in terms of the position of the roots of the corresponding characteristic equation towards the unit circle and in terms of the fractional orders of the considered Caputo-type differences and of the coefficients of the equation and the discretization step size. A possible application to the theoretical findings explored in the present work could be the analysis of the stability conditions in the discrete-time counterpart of the fractional-order Basset equation.

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Stability behaviour of a mathematical model which controls the unemployment and underemployment

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Presentation type: Online Talk

A five-dimensional mathematical model with two distributed time-delays for the control of unemployment and underemployment is built and analysed. We take into account five variables: the number of unemployed persons U, the number of immigrants M, the number of those underemployed or persons working limited hours T, the number of regularly employed persons R, and the number of available vacancies V. The proposed system allows for both job creation, as well as policy intervention to limit unemployment, but also the possibility that migrants are taking some of the regular positions. We deduce the non-dimensional system and we analyse the stability properties of the equilibrium points for the dynamical system, including

models with Dirac and weak Gamma kernels. We discretize the mathematical model in order to perform numerical simulations that highlight the theoretical results.

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Stable and Historic Behavior in Replicator Equations

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Presentation type: Talk

While studying the evolution of "*Binary Reaction System*" of three species which is a zero-sum game, S. Ulam conjectured the mean ergodicity of quadratic stochastic operators acting on the finite-dimensional simplex (see [11]). However, M. Zakharevich showed that Ulam's conjecture is generally false (see [12]). Later on, the complicated dynamics of "*Binary Reaction System*" which is known as *"historic behavior"* in the literature (see [4, 10]) was studied in great detail (see [1]). However, this is against a common belief (see [2, 3, 9]) that all *"reasonable"* replicator equations satisfy *"The Folk Theorem of Evolutionary Game Theory"* which asserts that (*i*) *a Nash equilibrium is a rest point*; (*ii*) *a stable rest point is a Nash equilibrium*; (*iii*) *a strictly Nash equilibrium is asymptotically stable*; (*iv*) *any interior convergent orbit evolves to a Nash equilibrium.* In this talk, we discuss two distinct classes of replicator equations which exhibit *stable* as well as *mean historic behavior.* In the latter case, the time averages of the orbit will slowly oscillate during the evolution and do not converge to any limit. This will eventually cause the non-existence of higher-order repeated time averages. This talk is based on the results published in the papers [5, 6, 7, 8].

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Stage-structure in interacting species models

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Presentation type: Talk

We consider two discrete-time models for interacting species that include stage structure in at least one species. The first model is a predator-prey system in which the prey species is modeled by juvenile and adult stages, and the predator is assumed to consume only newborn prey individuals. The second model considers a host-parasitoid system in which both species are modeled with stage structure. For both models, we obtain results pertaining to local and global stability and persistence. We compare the results for the predator-prey model to related models formulated under different stage structure assumptions. In addition, we demonstrate how the introduction of a predator can stabilize an unstable prey steady state. We also establish a bifurcation theorem which, when applied to the host-parasitoid model, determines the stability of the interior equilibrium near the bifurcation of the parasitoid-free equilibrium.

Stochastic fractional 2D-Stokes model with delay

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Presentation type: 'Talk'

The well-posedness of two versions of stochastic delay time fractional 2D-Stokes equations with nonlinear multiplicative noise are analyzed in this paper. The main tool to prove the existence and uniqueness of mild solution is based on fixed point arguments. The results for the first model can only be proved when $\alpha \in (1/2, 1)$, and the global existence in time has to be shown only when the noise is additive. As for the second model, all results are true for $\alpha \in (0, 1)$, and the global in time solutions are proved to exist for general nonlinear multiplicative noise. The analysis in the finite and infinite delay cases, although follow the same lines, require different phase spaces and estimations. Both cases are analyzed separately. This work can be considered as a first approximation to the challenging model of stochastic time fractional Navier-Stokes (with or without delay) which so far remains as an open problem.

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Monotonicity result for nabla fractional h-difference operators

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Presentation type: Talk

It is well known that monotonicity result plays an important role in the study of discrete fractional calculus, numerous monotonicity results about fractional calculus have been published. Until now, most of the monotonicity results of f, for $(\Delta_a^\nu f)(t)$, $(\nabla_a^\nu f)(t)$ are $\nu \in (1,2)$. But there are few monotonicity results of f are $\nu \in (0,1]$. There are just the ν-increasing (or ν-decreasing) results for $\nu \in (0,1)$, but they do not guarantee that these results hold for $\nu = 1$. In this paper, we give a new method to show the monotonicity result of a function f for $({}_a\nabla_h^\nu f)(t) \leq 0$ $(({}_a\nabla_{h,*}^\nu f)(t) \leq 0)$ with $\nu \in (0,1]$. In addition, we give an example to illustrate one of our main results.

The evolutionary stability of partial migration with Allee effects

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Presentation type: Talk

An Allee effect occurs when the per-capita growth rate increases at low densities. Here, we investigate the evolutionary stability of a partial migration population with migrant population experiencing Allee effects. Partial migration is a unique form of phenotypic diversity wherein migrant and non-migrant individuals coexist together. We will discuss that when Allee effect is incorporated, the population undergoes a bifurcation as the fraction of migrating population increases from zero to unity. Using an evolutionary game theoretic approach, we will talk about the existence of a unique evolutionary stable strategy (ESS). We will show that the ESS is the only ideal free distribution (IFD) that arises in the context of partially migrating population.

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THE LIMIT AS $p \to \infty$ FOR THE $p-$LAPLACIAN EQUATION WITH DYNAMICAL BOUNDARY CONDITIONS

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Presentation type: Talk

In this talk we study the limit as $p \to \infty$ in the evolution problem driven by the $p-$Laplacian with dynamical boundary conditions. We prove that the natural energy functionals associated with this problem converge in the sense of Mosco convergence to a limit functional and therefore we obtain convergence of the solutions to the evolution problems. For the limit problem we show an interpretation in terms of optimal mass transportation and provide examples of explicit solutions for some particular data.

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The Fundamental Theorem of Demography

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Presentation type: Talk

The difference equation $\mathbf{x}(t+1) = P\mathbf{x}(t)$, where P is an irreducible and nonnegative $n \times n$ matrix, arises as a discrete time model for demographic state variable $\mathbf{x}(t) \in \mathbb{R}^n_+$, $t = 0, 1, 2, \cdots$, associated with a biological population structured according to some classification scheme (by age, body size, life cycle stage, disease state, etc.) [1, 2, 3, 5, 6, 7]. If P is primitive, the strictly dominant, positive eigenvalue r of P is geometrically simple and has a positive eigenvector $\mathbf{v} \in \mathring{\mathbb{R}}^n_+$. The Fundamental Theorem of Demography (FTD) states that for all initial conditions $\mathbf{x}(0) \in \mathbb{R}^n_+ \setminus \{\mathbf{0}\}$: (a) $r < 1$ implies $\lim_{t\to\infty} \|\mathbf{x}(t)\| = 0$; (b) $r > 1$ implies $\lim_{t\to\infty} \|\mathbf{x}(t)\| = \infty$; and (c) in either case $\lim_{t\to\infty} \mathbf{x}(t) / \|\mathbf{x}(t)\| = \mathbf{v}/\|\mathbf{v}\|$ (the "stable distribution") [4]. (Here $\|\cdot\|$ is the L^1 norm, which is total population size in a population model.) Consequently, when $r > 1$ this linear difference equation cannot serve as a model for the long time dynamics of a population. Population models that regulate unbounded growth assume the entries in the projection matrix depends on $\mathbf{x}$. I will discuss the extent to which the properties of the FTD extend to nonlinear difference equations of the form $\mathbf{x}(t+1) = P(\mathbf{x}(t))\,\mathbf{x}(t)$.

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The importance of being (discrete) variational

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Presentation type: Talk

We discuss some recent developments and applications in the theory of variational difference equations [1, 7].

In particular we present a survey of the following results:

- a general solution of the inverse problem of the (discrete) calculus of variations for scalar equations of order $2k$, $k \geq 2$ [2],
- a classification of additive and multiplicative variational difference equation of order 4 [3, 4],
- a classification of integrable additive difference equations of order 4 possessing an invariant of a given form [3],
- application of the existence of variational structures in classifying discrete integrable system possessing coalgebra symmetry [5, 6].

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The numerical solution of the free-boundary cell motility problem

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Presentation type: Oral

The cell motility problem has been investigated for a long time, and today many biologists, physicists and mathematicians [2] are looking for new research instruments for this process. A simple 2D-model of a free-boundary cell moving on the homogeneous and isotropic surface was introduced in the paper [1]. It describes the dynamics of the complex actomyosin liquid, whose special properties impact the boundary dynamics and the cell motility. The model consists of a system of equations on the free boundary domain and containes a non-local term. Darcy law describes the flow of actomyosin liquid [3], and the myosin density in the cell changes according to the advection-diffusion equation. As for the boundary conditions, we assume that the cell membrane is described by Young-Laplace equation, the velocity of the liquid at the boundary is equal to the normal component of the cell membrane velocity and there is no flux of the myosin through the membrane.

$$\begin{cases} \Delta\varphi = \zeta\varphi - Q(m) \\ \frac{\partial m}{\partial t} + \nabla(m\nabla\varphi) = \Delta m \\ \zeta\varphi = \gamma\kappa + p_{eff}(|\Omega(t)|) \\ V_\nu = \partial\varphi \\ \partial_\nu m = 0 \end{cases}$$

where ϕ is a potential of the actomyosin liquid, m - a myosin density, t - time, ζ - an adhesion constant, γ - a surface tension coefficient, κ - a local curvature of the cell membrane, $p_{eff} = p_h + k_e(\frac{|\Omega_0|-|\Omega(t)|}{|\Omega_0|})$, p_h - a homeostatic pressure, k_e - a coefficient of elasticity, $|\Omega_0|$ - a value of the stationary area of the cell, $|\Omega(t)|$ - an area of the cell at t, V - a velocity of the membrane, ν - a normal vector of the membrane, Q(m) - a myosin stress function.

A numerical algorithm was developed to obtain a approximate solution of the problem. To do that, the coordinate system was changed to the curvilinear polar coordinates and problem was reduced to the boundary value one with a fixed border [4]. The numerical scheme of the second order of accuracy was developed using the scheme described in [5] for the finite difference method on the uneven grid. Stable solutions that coincide with the analytical ones were obtained with the asymptotic accuracy verified. The travelling wave solutions, predicted in the paper [1], were observed in our numerical experiments.

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The real teapot

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Presentation type: Talk

William Thurston in his last paper defined a Master Teapot as the closure of the set of pairs (s, z), where $s \in (1, 2]$ is the slope of a tent map T_s with the turning point periodic, and a complex number z is a Galois conjugate of s. Then $1/s$ and $1/z$ are zeros of the kneading determinant of T_s. We study the zeros of the kneading determinants of the maps T_s that belong to the real interval $(0, 1)$, for all $s \in (1, 2]$.

This is work in progress.

Theoretical and practical aspects for discrete optimal partitioning problems

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Optimal partitioning problems regarding the eigenvalues of the Dirichlet-Laplace operator appear naturally when studying various models in population dynamics, chemical reactions and clustering problems. The search for partitions with n cells of a fixed domain D which minimize the sum of the fundamental eigenvalues has generated many recent works. Often, these optimal partitions do not have a precise theoretical characterization, motivating the development of numerical algorithms in [1], [2].

In this presentation, the discrete numerical algorithms mentioned above are formulated using a direct recurrence relation inspired from the gradient descent algorithm with projection. New results concerning the convergence of these numerical algorithms are presented, together with practical illustrations.

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Triple solutions for a Dirichlet boundary value problem involving a perturbed fractional operator

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Presentation type: Talk

Abstract

Triple solutions are obtained for a discrete problem satisfying Dirichlet boundary conditions and involving a nonlinearly perturbed one-dimensional operator and the nabla left and right Gerasimov-Caputo and Riemann-Liouville fractional differences. The methods for existence rely on a Ricceri-local minimum theorem for differentiable functionals. an examples is included to illustrate the main results.

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Viscoelasticity and magneto-viscoelasticity: an overview on recent results

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Presentation type: 'Talk'

The model of viscoelastic body is reconsidered. In particular, for sake of simplicity, a one dimensional case is analysed. The model equation is an integro-differential one whose kernel represents the relaxation function G. The attention is focussed on the relaxation function G which describes the mechanical behaviour of the material. Indeed, the relaxation function represents the mathematical tool to model the dependence of the mechanical behaviour of the material on time not only via the present time but also on its past history. Based on the classical viscoelasticity model existence and uniqueness of the solution given by Dafermos [3] can be generalised to consider materials *non classical* viscoelastic models [4, 1, 7] in which the relaxation function G is unbounded at the origin or less regular [6] or *aging* effects [5] are taken into account. Also the combined effects of magnetic sensibility with viscoelastic behaviour is of interest [2]. Indeed, micro, or nano, particles are inserted in viscoelastic materials to modify their mechanical response on application of an external magnetic field.

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Weakly nonwandaring points in dynamics of skew products in high dimensions

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Presentation type: 'Talk'

The concept of weakly nonwandering points with respect to the family of fibre maps over wandering points of the quotient of a skew product in the plane is introduced in [1]. Here we consider continuous skew products on n-dimensional ($n \geq 2$) manifolds such as cells, cylinders and tori, and generalize the concept of these points on the multidimensional case. We prove the criterion for existence of weakly nonwandering points with respect to the family of fibre maps over wandering points of the quotient in the terms of Ω-blow up (in the C^0-norm) in fibre maps over limit (for wandering set) nonwandering points of the quotient.

Using these results we describe the structure of the nonwandering set of skew products on n-dimensional cells, cylinders and tori under the following conditions:

(1) the set of periodic points is not empty (for self-maps of cylinders and tori), and it is closed;

(2) the set of periodic points is empty for self-maps of cylinders and tori, and the nonwandering set is minimal (see [2]).

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Well-Posedness of Nonlinear Integro-Dynamic Equations on Time Scales

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Presentation type: Talk

This paper considers a nonlinear integro-dynamic equation on time scales with the local initial condition. This paper aims to prove the existence and uniqueness of solutions and investigate qualitative properties of solutions of this equation such as boundedness, and dependence of solutions on initial conditions, functions, and parameters. The primary tool used here is Gronwall-type dynamic inequalities. Examples are provided to illustrate the results.

p-Homogenization of Fractal Membranes

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Presentation type: Talk

In this talk we consider a quasi-linear homogenization problem in a two-dimensional pre-fractal domain Ω_n, for $n \in \mathbb{N}$, surrounded by thick fibers of amplitude ε. We introduce a sequence of "pre-homogenized" energy functionals and we prove that this sequence converges in a suitable sense to a quasi-linear fractal energy functional involving a p-energy on the fractal boundary, thus generalizing the results known for the linear fractal case. We then prove existence and uniqueness results for (quasi-linear) pre-homogenized and homogenized fractal problems and we investigate the convergence of the pre-fractal solutions to the limit fractal one.

XII. Author Index

Author Index